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Articles

Comparison of the Standardized Palmer Drought Index (SPDI) in three climatic locations in San Luis Potosi, Mexico

Contrastes del Índice de Sequías de Palmer Estandarizado (SPDI) en tres ubicaciones climáticas de San Luis Potosí, México

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Abstract

Meteorological Droughts are extreme natural phenomena with decreased precipitation, and thus constitute a threat to ecosystems and human societies. *Drought Indices* are detection and monitoring tools. These indices make use of climatic variables in order to obtain a characteristic numerical value that is more useful than the original raw data. The SPDI (Standardized Palmer Drought Index) is a recently proposed multivariate-type index, which uses a soil water balance with potential evapotranspiration estimated using the Thornthwaite method and a probabilistic approach to process *moisture deviations* for different durations of drought. The SPDI was applied to monthly precipitation and average temperature data from three climatological stations with large registries (≥ 50 years) from the state of San Luis Potosí, Mexico, located in each of its three geographical areas: Altiplano Potosino, Middle Zone and Huasteca Region. These stations are: Villa de Arriaga, Río Verde and Xilitla. A contrast between the SPDI results and those calculated with the two preceding indices (SPI and SPEI) was carried out for each of these climatological stations. The former uses only monthly precipitation data and the latter incorporates potential evapotranspiration. The aforementioned comparison covers drought durations of 6, 12 and 24 months. Results indicate that the percentages of each type of drought indicated by the three indices have similar values in terms of the order of magnitude; however, they differ in a specific way for each drought duration, mainly in relation to severe and extreme droughts. The evolution graphs of the SPDI make it possible to clearly highlight periods of drought, defining their start and end dates as well as maximum events including both severity and the date of occurrence.

Keywords: Meteorological droughts, soil water balance, potential evapotranspiration, statistical tests, moving sums, Log-Logistic distribution, SPI and SPEI indices, types of droughts.

Resumen

Las *sequías meteorológicas* son fenómenos naturales extremos que originan una precipitación menor que la normal, y por ello constituyen una amenaza para los ecosistemas y las sociedades humanas. Para su detección y monitoreo se emplean los *índices de sequía*, los cuales emplean variables climáticas, a fin de obtener un valor numérico que las caracterice y sea más útil que los datos originales. El SPDI (Standardized Palmer Drought Index) es un índice propuesto recientemente, de tipo multivariado, que utiliza un balance hídrico edafológico, con evapotranspiración potencial estimada, con el método de Thornthwaite y un enfoque probabilístico para procesar las *desviaciones de humedad* en diferentes duraciones de sequía. El SPDI se aplicó a los datos mensuales de precipitación y temperatura media de tres estaciones climatológicas de amplio registro (≥ 50 años) del estado de San Luis Potosí, México, que se ubican en cada una de sus tres áreas geográficas: Altiplano Potosino, Zona Media y Región Huasteca. Tales estaciones son las siguientes: Villa de Arriaga, Río Verde y Xilitla. En cada una de estas estaciones climatológicas se realizó un contraste de resultados del SPDI, contra los que se calculan con los dos índices predecesores el SPI y SPEI. El primero emplea sólo datos de precipitación mensual y el segundo incorpora a la evapotranspiración potencial. El contraste citado se realizó para tres duraciones de sequía de 6, 12 y 24 meses. Los resultados indican que los porcentajes de cada tipo de sequía que establecen los tres índices contrastados son valores similares en orden de magnitud, pero que difieren de manera puntual para cada duración de sequía, primordialmente en relación con las sequías severas y extremas. Por otra parte, las gráficas de evolución del SPDI permiten observar de manera clara los periodos de sequía, definiendo sus fechas de inicio y terminación, así como sus eventos máximos tanto en severidad como en fecha de ocurrencia.

Palabras clave: sequías meteorológicas, balance hídrico edafológico, evapotranspiración potencial, pruebas estadísticas, sumas móviles, distribución Log-Logística, índices SPI y SPEI, tipos de sequías.

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Introduction

Definitions

Meteorological droughts (MD) are recurrent extreme climatic events that occur in any area of the country, including humid regions. They are characterized by scarce or lower than normal precipitation, occurring for several months or years. MDs are dry periods, as opposed to the permanent dryness of arid regions. Due to the constant increase in water demand for all uses and the negative effects of climate change, *droughts* have become more severe and their impacts are more noticeable and prolonged (Mishra & Singh, 2010; Fuchs, Svoboda, Wilhite, & Hayes, 2014).

Given that in some water systems the period between the shortage of precipitation and its detection as a *deficit* varies considerably, droughts have been established as a phenomenon on multiple scales over time. *Meteorological*, *agricultural*, *hydrological* and *socio-economic droughts* are defined when deficits occur in rainfed agriculture, irrigation, generation of hydroelectric energy, municipal and industrial water supply, as well as when economic, social and environmental impacts are quantified (Pandey, Sharma, Mishra, Singh, & Agarwal, 2008; Vicente-Serrano, Beguería, & López-Moreno, 2010; Fuchs *et al*., 2014).

In general, MDs are a regional phenomenon characterized by three dimensions: severity or intensity, duration and surface extension (Tsakiris & Vangelis, 2005). This has led to the development of various drought indices for their detection and monitoring, which analyze separately or jointly the dimensions cited and allow the objective comparison between MDs that occur in dissimilar climates and help in the formulation of mitigation plans of their negative effects (Pandey *et al*., 2008; Vicente-Serrano *et al*., 2010; Fuchs *et al*., 2014).

Common MD Indices

The first meteorological drought index that has been broadly used both in the U.S.A. as well as in other countries was the PDSI (Palmer Drought Severity Index), presented in the mid 1960s (Palmer, 1965), so it is over 50 years old. It is a multivariate index that applies a soil moisture balance (Hao & Singh, 2015) and uses monthly potential evapotranspiration and precipitation. But it is based on several fully empirical rules, as demonstrated by Alley (1984), and is quite sensitive to the period used for its coefficient calibration, as shown by Karl (1986). It also does not allow the use of multiple time scales or drought durations. Wells, Goddard and Hayes (2004) developed a procedure for automatic calculation of the PDSI, which improves the regional comparison capacity of the index. Pereira, Rosa and Paulo (2007) used another water balance model to adapt the PDSI to the Mediterranean climate.

The next meteorological drought index with nearly universal application is the SPI (Standardized Precipitation Index), proposed in the early 1990s (McKee, Doesken, & Kleist, 1993). This is multi-scalar in time and uses a probabilistic approach to analyze monthly precipitation, which has proven to be quite efficient. Such approach consists of two processes. The first analyzes monthly precipitation sequences as *moving sums*, according to the study *duration* of the droughts, which ranges from 1 to 72 months. In the second process, the formed sequences are fitted with a *probability distribution function* (FDP) to estimate their probability of non-exceedance, which by means of a numerical approximation is transformed into a standard normal variable (Z). The negative values correspond to meteorological drought sequences and their types are defined as light, moderate, severe and extreme, with the value of Z = SPI ranging from 0 to -1.00, from -1.00 to -1.50, from -1.50 to -2.00 and when it is less than -2.00, respectively.

Recently, the SPEI (Standardized Precipitation Evapotranspiration Index) has been proposed by Vicente-Serrano *et al*. (2010) and Beguería, Vicente-Serrano, Reig and Latorre (2014), which has a calculation similar to that of SPI, but uses a measure of the monthly climate balance based on the difference (d) between monthly precipitation and potential evapotranspiration, instead of just precipitation. Because the differences, d, are generally negative, some moving sums or sequences are less than zero, in which case the FDP used has three fitting parameters, and that of location (u) must be less than the minimum sequence. The creators of the SPEI found that the log-logistics FDP results in a good probabilistic modeling of the sequences. Stagge, Tallaksen, Gudmundsson, Van Loon and Stahl (2015) have suggested the FDP General Extreme Values.

An important advantage of the SPEI over the SPI lies in incorporating the potential evapotranspiration variable and the ability to consider possible effects of climate change on the severity of meteorological droughts, through modifying or altering the historical precipitation and average monthly temperature records by reducing the former and/or increasing the latter, as shown by Vicente-Serrano *et al*. (2010).

Ma *et al*. (2014) described deficiencies and limitations in the difference, d, used by the SPEI as an equation of the monthly climate balance and proposed the use of the calculated *moisture deviation*, $\hat{d}$, as named by Palmer (1965). The latter is equivalent to the difference between the observed precipitation and precipitation estimated for normal climate conditions in the region ($\hat{P}$), which is calculated based on the results of a soil water balance. Ma *et al*. (2014) found that d and $\hat{d}$ respond differently to variations in precipitation and temperature, according to climatic conditions. For dry climates, they found that d-series are excessively dependent on temperature, leading to large and even abnormal deficits as potential evapotranspiration grows, which is unattainable as real evapotranspiration in times of low precipitation.

Fortunately, the difference, $\hat{d}$, shows low variability in relation to temperature while also retaining the influence of precipitation, and therefore it has better spatial behavior. Ma *et al*. (2014) processed the differences, $\hat{d}$, with the SPI probabilistic approach, to develop the Standardized Palmer Drought Index (SPDI). This index includes the theoretical salvageable part of the Palmer index, the computational efficiency of the SPI and reproduces the sensitivity of the SPEI to study probable climate changes.

Objective

This study has three objectives: (1) the operating procedure of the SPDI is presented in detail, which was recently proposed by Ma *et al*. (2014); (2) SPDI results are compared to the preceding indices (SPI and SPEI) for three MD durations of 6, 12 and 24 months, which attempt to characterize the agricultural and hydrological droughts; (3) these comparisons are carried out in three climatological stations (Villa de Arriaga, Río Verde and Xilitla) in the state of San Luis Potosí, Mexico, which have extensive records (≥ 50 years) and are located in each of their three geographical regions: Potosino Plateau, Middle region, and Huasteca Region. Based on the results of the analysis, conclusions are formulated.

Materials and methods

Soil water balance

The original Palmer index (PDSI) starts with a *soil water balance* (SWB), performed on a monthly basis using historical precipitation (*P*) and average temperature (*Tt*) records. The storage (*S*) of moisture in the soil occurs in two layers. The first surface can save (Ss_{max}) up to 25 millimeters in water depth and the second, which is deeper, has a capacity (Su_{max}) that depends on the physical characteristics of the soil and the depth of the roots of vegetation, ranging from 127 to 229 millimeters (Palmer, 1965). Figure 1 shows the flow chart of the SWB.

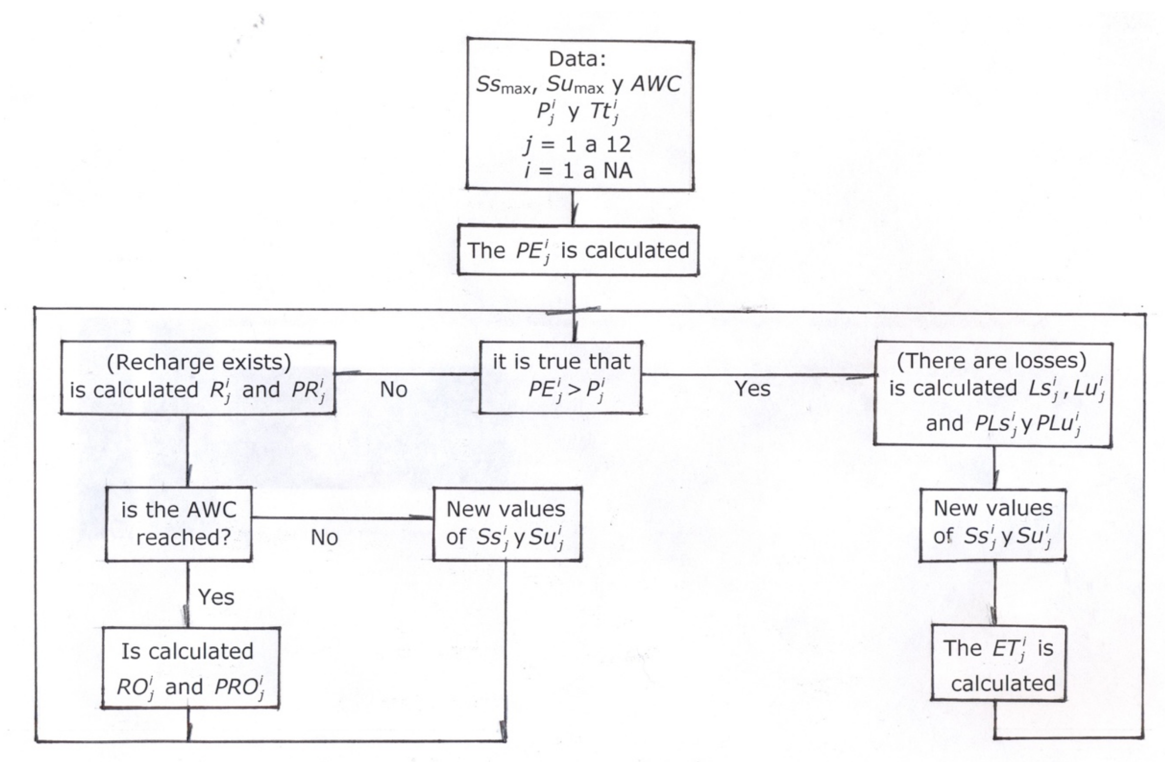

Figure 1. Flow chart of the water soil balance of the Palmer index (PDSI).

Moisture cannot be extracted (or stored) in the deep layer (*Su*) until all the available moisture in the surface layer (*Ss*) is taken (or recharged). Potential evapotranspiration (*PE*) is estimated with the Thornthwaite method, which is presented next. The losses from soil (*L*) due to *PE* occur when *PE*> *P*, where *P* is precipitation. Evapotranspiration losses from the surface layer (*Ls*) occur at a potential level, while those from the deep layer (*Lu*) depend on the initial moisture content, *PE* and the storage moisture in both soil layers, designated by *AWC* (available water capacity = Ss_{max} + Su_{max}). Then, when *PE* > *P*, we have (Palmer, 1965; Alley, 1984):

$$Ls = mínimo\ [Ss, (PE - P)] \qquad (1)$$

$$Lu = \frac{Su \cdot [(PE - P) - Ls]}{AWC} \quad \text{siendo} \quad Lu \leq Su \qquad (2)$$

In the previous equations, *Ss* and *Su* are the moisture contents stored in each soil layer at the beginning of the respective month. The SWB of the Palmer method considers runoff (*RO*) to only occurs when the two layers of soil reach their total storage capacity (*AWC*) and is equivalent to the surplus of *P* minus the *PE* (availability) in the month, after filling both soil layers (Δ*Ss* and Δ*Su*).

$$RO = (P - PE) - \Delta Ss - \Delta Su \qquad (3)$$

The concept of *PE* only occurs, according to its definition, when vegetation is actively growing, so that in arid climates it will not occur during the dry season, and in cold climates it will not occur during the winter. Then, the current or real evapotranspiration (*ET*) occurs when *PE* > *P* and is equal to:

$$ET = P + (Ls + Lu) \qquad (4)$$

Climatically normal monthly precipitation

As part of the SWB of the Palmer method, three additional terms are calculated with a potential or maximum level: recharge (*PR*), losses (*PL*) and runoff (*PRO*). The potential recharge for the month is defined as the amount of water necessary to bring the soil to its field capacity, therefore, this is equal to (Palmer, 1965; Alley, 1984):

$$PR = AWC - (Ss + Su) \quad (5)$$

where *Ss* and *Su* are the values from the previous month. Potential losses are considered equal to the amount of moisture that can be lost from the soil due to *PE* when there is no precipitation; therefore they are:

$$PL = PLs + PLu \quad (6)$$

$$PLs = mínimo\ (PE, Ss) \quad (7)$$

$$PLu = \frac{Su \cdot (PE - Pls)}{AWC} \quad \text{siendo} \quad Plu \leq Su \quad (8)$$

Finally, regarding the potential runoff (*PRO*), potential precipitation (*PP*) is assigned to this value, which Palmer (1965) assigns to the value of *AWC*, less than potential recharge (*PR*). For the above:

$$PRO = AWC - PR \quad (9)$$

In the previous equation, originally used by Palmer (1965), it is obvious that the potential precipitation (*PP*) has a physically weak or scarce relation with the assigned value of *AWC* and therefore Palmer later suggested using the *PP*, equal to three times the average monthly precipitation ($\bar{P}_j$). This modification was taken into account by Pereira *et al*. (2007).

The four calculated potential values (*PE*, *PR*, *PL* and *PRO*) are used to calculate four coefficients (alpha, beta, gamma and delta) that are quotients of average values, which depend on the climate of the area that is being studied. They are:

$$\alpha_j = \frac{\overline{ET}_j}{\overline{PE}_j} \qquad (10)$$

$$\beta_j = \frac{\overline{R}_j}{\overline{PR}_j} \qquad (11)$$

$$\gamma_j = \frac{\overline{RO}_j}{\overline{PRO}_j} \qquad (12)$$

$$\delta_j = \frac{\overline{L}_j}{\overline{PL}_j} \qquad (13)$$

where *j* ranges from 1 to 12, representing the month counter, and the average quantities are calculated with *i* annual values, which go from 1 to NY number of years on record (> 30). The above *SWB coefficients* are used to calculate CAFEC precipitation (climatically appropriate for existing conditions, named by Palmer (1965) designated by $\hat{P}$, that is:

$$\hat{P}_j^i = \alpha_j \cdot PE_j^i + \beta_j \cdot PR_j^i + \gamma_j \cdot PRO_j^i - \delta_j \cdot PL_j^i \qquad (14)$$

The precipitation, $\hat{P}$, that is *climatically appropriate for the existing conditions* is an estimate from the SWB, which is equivalent, according to Ma *et al*. (2014), to the minimum amount of precipitation necessary to maintain soil moisture under *normal conditions* in a certain region. According to Equation 14, it corresponds to the sum of real evapotranspiration, groundwater recharge and direct runoff, minus the change in moisture storage in the soil (Palmer, 1965; Alley, 1984).

Moisture Deviation

The differences, $\hat{d}$, between monthly precipitation P and $\hat{P}$ in each month recorded are called *moisture deviation* (moisture departure) and corresponds to (Palmer, 1965):

$$\hat{d}_j^i = P_j^i - \hat{P}_j^i \qquad (15)$$

As indicated, Ma *et al*. (2014) processed the differences, $\hat{d}$, with the probabilistic SPI approach to develop the SPDI.

Monthly Potential Evapotranspiration

The Thornthwaite criterion estimates the PE_j^i, in millimeters, of month j of year i, with the following empirical equation (Mather, 1977; Campos-Aranda, 2005; Xu, Singh, Chen, & Chen 2008):

$$PE_j^i = 16 \cdot Fc \cdot \left(\frac{10 \cdot Tc_j^i}{IC_i}\right)^m \qquad (16)$$

where Fc is a corrective factor function of the latitude of the site and ndm is the number of days of the month. Its formula is:

$$Fc = \left(\frac{N}{12}\right) \cdot \left(\frac{ndm}{30}\right) \qquad (17)$$

where N is the maximum sunlight or maximum number of hours with average monthly sunshine. For its estimation in the Mexican Republic, Campos-Aranda (2005) proposed the following empirical expression:

$$N = A + B\,[sen\,(30\,nm + 83.5)] \qquad (18)$$

where *nm* is the number of the month, with 1 for January and 12 for December; *A* and *B* are constants functions of the latitude (LAT) of the site, in degrees, with the following expressions:

$$A = 12.09086 + 0.00266 \cdot LAT \qquad (19)$$

$$B = 0.2194 - 0.06988 \cdot LAT \qquad (20)$$

In Equation (16), Tt_j^i is the monthly average temperature in °C, IC_i is an annual heat index, equal to the sum of the 12 monthly indices, which are:

$$icm = \left(\frac{Tt_j^i}{5}\right)^{1.514} \qquad (21)$$

Finally, in Equation 16 the exponent *m* is a function of IC_i with the following empirical expression:

$$m = 6.75 \cdot 10^{-7} \cdot IC_i^3 - 7.71 \cdot 10^{-5} \cdot IC_i^2 + 1.792 \cdot 10^{-2} \cdot IC_i + 0.4924 \qquad (22)$$

For values of Tt_j^i greater than 26.5 °C the IC_i has no influence, so the PE_j^i is only a function of Tt_j^i and is tabulated (Campos-Aranda, 2005). The tabulation values cited can be represented by a line when Tt_j^i varies from 26.5 to 28.0 °C and by a second-degree polynomial when Tt_j^i is greater. These equations are:

$$PE_j^i = -90.4106 + 8.5114 \cdot Tt_j^i \qquad (23)$$

$$PE_j^i = -423.7983 + 32.7289 \cdot Tt_j^i - 0.43989 \cdot \left(Tt_j^i\right)^2 \qquad (24)$$

Fitting of the Log-Logistic Distribution

Having calculated the differences, $\hat{d}$, with Equation 14, a *chronological series* is obtained of 12 by NY elements, being NY the number of years in the record processed. Then, these series are used to form sequences, such as *moving sums* of size *k* in months, equal to the *duration of the drought* that is being studied, commonly the following: 1, 3, 6, 9, 12, 18, 24, 30 and 36 months. For example, for $k = 3$ the first sequence will be the sum of the first, second and third value of $\hat{d}$, the second sequence will be the sum of the second, third and fourth value of $\hat{d}$ and so on until the last sequence (*ns*) that will be formed with the sum of the antepenultimate, penultimate and last value of $\hat{d}$. Therefore, the number of sequences (*ns*) that can be formed will be equal to:

$$ns = 12 \cdot NA - k + 1 \qquad (25)$$

The formed sequences will be designated by D_l^k with *l* varying from one to *ns*. For the fitting of an FDP to the formed sequences, Ma *et al*. (2014) tested four probabilistic models: log-normal, Pearson type III, log-logistic and generalized extreme value distribution (GVE), recommending the latter, fitted with the L-moments method. For the case of the climatic information processed in this study, the use of the GVE distribution did not lead to a fitting that fulfills the condition that the location parameter (*u*) be lower than the magnitude of the minimum sequence ($D_{\min}^k$), therefore, the FDP log-logistics (LL3) was used, suggested by Vicente-Serrano *et al*. (2010) and verified by Beguería *et al*. (2014) for its use with the SPEI. The LL3 function is equivalent to the generalized logistic distribution through an adequate relation of parameters (Rao & Hamed, 2000). The equation for the accumulated probability distribution function [$F(x)$] of the LL3 distribution is (Haktanir, 1991):

$$F(x) = \left[1 + \left(\frac{x-u}{\alpha}\right)^{-1/\gamma}\right]^{-1} \quad (26)$$

where $\gamma > 0$, $\alpha > 0$ and $u < x_{mo}$ are the parameters of form, scale and location, whose values can be estimated with various statistical procedures. x_{mo} is the minimum observed datum ($D^k_{\min}$). Ahmad, Sinclair and Werritty (1988) found that the probability-weighted moments (PWM) method (βs), developed by Greenwood, Landwehr, Matalas and Wallis (1979), is the most robust, as well as being a relatively simple and direct procedure. According to Stedinger, Vogel and Foufoula-Georgiou (1993), the biased estimators of β_s must be used when fitting an FDP at a single site, and quantiles are estimated with it, because then their predictions have a lower mean squared error. These moments are calculated with the following two expressions (Hosking, 1990; Haktanir, 1991; Stedinger *et al*., 1993; Vicente-Serrano *et al*., 2010:

$$\beta_s = \frac{1}{ns}\sum_{l=1}^{ns}(1-F_l)^s \cdot D^k_l \qquad \text{con s = 0, 1, 2, …} \quad (27)$$

where:

$$F_l = \frac{l-0.35}{ns} \quad (28)$$

where l is the counter of sequences whose number is ns. D^k_l are the sequences in an ascending order $\left(D^k_1 \le D^k_2 \le \cdots \le D^k_{ns}\right)$. Beguería *et al*. (2014) have suggested using the unbiased βs when the biased estimators do not lead to a numerical solution. The equations for estimating the three fitting parameters of the LL3 distribution are (Haktanir, 1991):

$$\gamma = 3 - \frac{2\cdot(\beta_0 - 3\cdot\beta_2)}{(\beta_0 - 2\cdot\beta_1)} \quad (29)$$

$$\alpha = \frac{(\beta_0 - 2 \cdot \beta_1)}{\gamma \cdot \Gamma(1+\gamma) \cdot \Gamma(1-\gamma)} \quad (30)$$

$$u = \beta_0 - \alpha \cdot \Gamma(1+\gamma) \cdot \Gamma(1-\gamma) \quad (31)$$

where Γ(·) is the factorial function Gamma, which was estimated with the Stirling formula (Davis, 1972), this is:

$$\Gamma(\varepsilon) = e^{-\varepsilon} \cdot \varepsilon^{\varepsilon-1/2} \sqrt{2\mu} \cdot \left(1 + \frac{1}{12\varepsilon} + \frac{1}{288\varepsilon^2} - \frac{139}{51840\varepsilon^3} - \frac{571}{2488320\varepsilon^4} \cdots\right) \quad (32)$$

Having calculated the three fitting parameters (*u*, σ, λ), Equation 26 is applied with $x = D_l^k$ to estimate the probabilities of non-exceedance *F*(*x*) that correspond to each moving sum of duration *k*.

Calculation of the SPDI Values

Finally, in accordance with McKee *et al*. (1993), the rational numerical approximation developed by Zelen and Severo (1972) is used to convert *F*(*x*) into the normalized standard variable Z of zero mean and unit variance, which defines the SPDI that is sought; their equations are:

$$Z = \text{SPI} = -\left(t - \frac{b_0 + b_1 \cdot t + b_2 \cdot t^2}{1 + c_1 \cdot t + c_2 \cdot t^2 + c_3 \cdot t^3}\right) \quad \text{para} \quad 0 < F(x) < 0.50 \quad (33)$$

$$Z = \text{SPI} = -\left(t - \frac{b_0 + b_1 \cdot t + b_2 \cdot t^2}{1 + c_1 \cdot t + c_2 \cdot t^2 + c_3 \cdot t^3}\right) \quad \text{para} \quad 0.50 < F(x) < 1 \quad (34)$$

where:

$$t = \sqrt{\ln\left[\frac{1}{[F(x)]^2}\right]} \quad \text{para} \quad 0 < F(x) < 0.50 \qquad (35)$$

$$t = \sqrt{\ln\left[\frac{1}{[1-F(x)]^2}\right]} \quad \text{para} \quad 0.50 < F(x) < 1 \qquad (36)$$

$b0 = 2.515517\; b1 = 0.802853\; b2 = 0.010328$

$c1 = 1.432788\; c2 = 0.189269\; c3 = 0.001308$

Processed Climatic Records

The state of San Luis Potosí is divided by their common watershed dividing Hydrological Regions No. 37 (El Salado) and No. 26 (Pánuco), as shown in Figure 2. The former region has an arid climate in the area known as Altiplano Potosino. In contrast, the Potosino portion of the Pánuco River basin is hot-humid; it is known as the Huasteca Region and begins approximately at the meridian 99° 30' W.G. This transition of climates creates a third geographic zone in the state, called Middle Zone, which has a temperate climate.

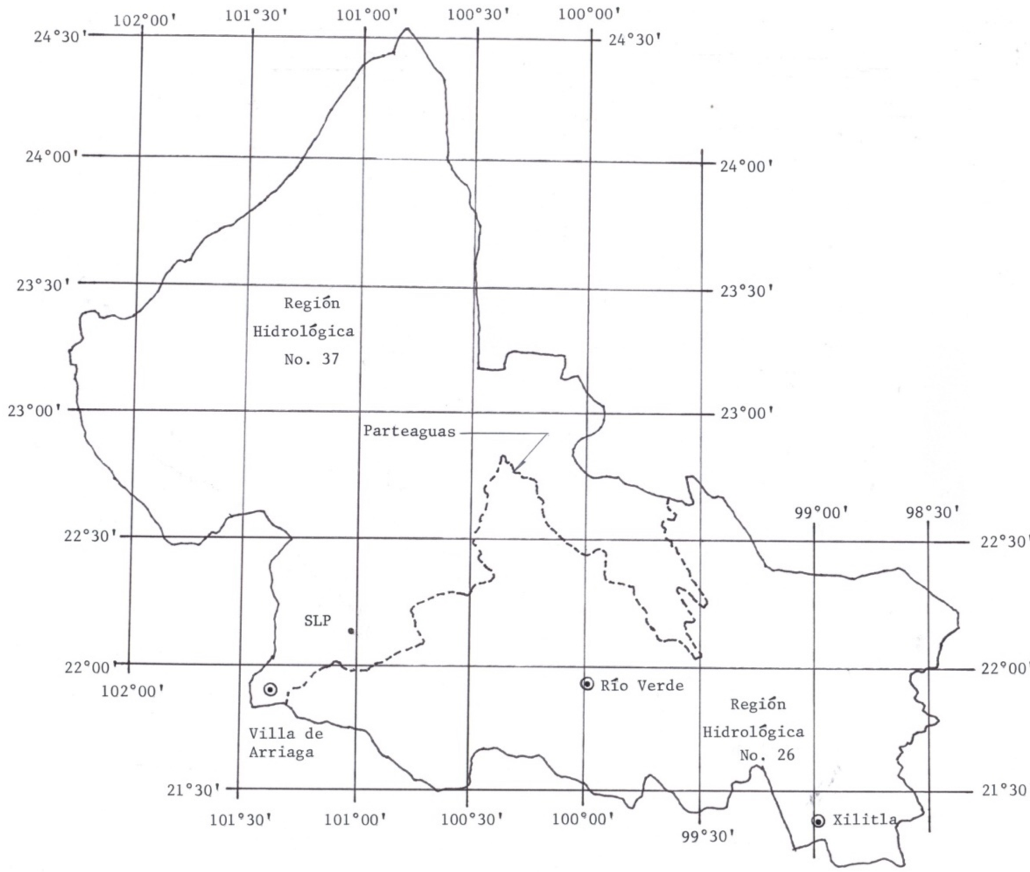

Figure 2. Geographic location of the three climatological stations processed in the state of San Luis Potosí, Mexico.

In each of these three geographical zones, climatological stations were identified with largest number of records and the least number of missing monthly data of precipitation and average temperature. The following three stations were selected: Villa de Arriaga, Río Verde and Xilitla. In each of them, the few missing rainfall data were considered equal to the monthly *mode*, estimated based on the fitting of the Mixed Gamma distribution to all available monthly values (Campos-Aranda, 2005). The few missing average temperature data were estimated with an interpolation procedure that took into account the trend observed in the month before and after the missing value.

The *average monthly* precipitation (mm) and average temperature (°C) values of each climatological station studied are listed in Table 1, as well as the respective recording periods. At the Villa de Arriaga station in the period from 2010 to 2014, average monthly temperature values were used, because its available records covered until 2009, when they were processed by Campos-Aranda (2017). Table 1 shows the monthly average calculated potential evapotranspiration (PE_j), based on the Thornthwaite method (Equations (16) to (24)). Figure 2 shows the location of the three climatological stations studied in the state of San Luis Potosí, Mexico.

Table 1. Average monthly values of the climatic elements indicated in the three climatological stations processed in the state of San Luis Potosí, México.

Description:	Jan	Feb	Mar	Apr	May	Jun	Jul	Aug	Sep	Oct	Nov	Dec	Anual
Station: Villa de Arriaga [1962-2014] (Longitude 101° 23' WG. Latitude 21° 54' N. Altitude 2170 masl).													
Precipitation	13.0	7.6	7.0	10.4	30.9	57.2	72.1	56.8	62.7	25.4	5.6	8.9	357.7
Mean Temp.	13.0	13.9	15.9	19.2	20.9	20.8	19.8	19.5	18.8	16.6	14.6	13.4	17.2
PE_i	37.6	39.3	57.6	82.1	102.3	99.4	94.1	87.9	77.1	60.1	44.7	39.1	821.2
Station: Río Verde [1961-2014] (Longitude 99° 59' WG. Latitude 21° 56' N. Altitude 987 masl).													
Precipitation	12.2	10.8	9.4	32.7	36.5	88.7	88.3	71.7	103.4	44.2	15.4	12.9	526.2
Mean Temp.	16.2	18.3	21.7	24.6	26.4	26.1	25.0	25.1	23.9	21.8	19.0	17.0	22.1
PE_i	36.6	47.5	84.3	118.5	142.6	137.2	134.6	130.3	107.1	82.6	54.3	41.0	1118.6
Station: Xilitla [1965-2014] (Longitude 98° 59' wG. Latitude 21° 23' N. Altitude 630 masl).													
Precipitation	62.6	65.3	72.5	115.3	175.5	373.9	432.2	429.9	566.1	292.5	101.5	59.0	2746.2
Mean Temp.	17.4	18.7	21.4	24.2	25.9	26.2	25.6	25.9	25.0	23.1	20.3	18.3	22.7
PE_i	42.1	48.2	80.2	113.2	137.0	139.8	137.3	138.4	118.8	94.9	62.6	47.2	1159.5

Results

Homogeneity Tests Applied

Based on the complete precipitation and average monthly temperature records, annual values were calculated. Based on these *series*, a statistical quality analysis was performed, for which the following seven tests were applied, one general and six specific: (1) Von Neumann, which detects decreased randomness by unspecified deterministic components, (2) Anderson and (3) Sneyers, which identify persistence, (4) Kendall and (5) Spearman, which detect trend, (6) Bartlett, which tests variability and (7) Cramer to identify changes in the mean. In all tests, a level of significance (α) of 5% was used. The statistical tests cited can be found in WMO (1971), Buishand (1982) and Machiwal and Jha (2008). The results of these tests are shown in Table 2, in which NH and H mean non-homogeneous and homogeneous series or record, respectively.

Table 2. Results of the statistical tests applied to annual precipitation (*P*) and average temperatures (*Tt*) records from the three climatological stations studied.

Statistical tests:	**Villa de Arriaga**		**Río Verde**		**Xilitla**	
	P	***Tt***	***P***	***Tt***	***P***	***Tt***
1. Von Neumann	NH	NH	NH	NH	H	NH
2. Persistence (Anderson)	NH	NH	NH	NH	H	NH
3. Persistence (Sneyers)	NH	NH	NH	NH	H	NH
4. Trend (Kendall)	H	NH↑	H	NH↑	H	NH↑
5. Trend (Spearman)	H	NH↑	H	NH↑	H	NH↑
6. Variability (Bartlett)	H	NH	H	H	H	H
7. Change in the mean (Cramer)	H	NH	H	NH	H	NH

Regarding the annual precipitation (*P*), the records from Villa de Arriaga and Río Verde show persistence, detected even with the von Neumann test. Since persistence is a statistical component of the time series, analyses aimed at quantifying meteorological droughts can continue, given that the three records show no trend or changes in the mean, that is, loss of homogeneity.

The opposite occurred with annual average temperature (*Tt*) records, which are totally non-homogeneous since they present persistence, an upward trend and a change in the mean. Accepting that this loss of homogeneity is associated with climate change, these records can be processed, and the effects of the upward trend will be reflected in the monthly estimates of potential evapotranspiration, with Equations 16 to 24.

Relevant aspects of the computer program

The program designated by **SPDI3** consists of three modules that were generated successively and thus were incorporated. In subprogram **SPDI1**, the SWB was developed based on Equations 1 to 15, which was completed when it succeeded in reproducing the values in Table 1 on page 10 of Palmer's paper (1965), which presents three years (1933 to 1935) of a SWB performed in Central Iowa using Ss_{max} and Su_{max} equal to 1.0 and 9.0 inches, and starting with *AWC* equal to 10.0 inches. This module, called SWB, concludes with the calculation of moisture deviation ($\hat{d}$) with Equation 15, in metric decimal system; estimating the potential evapotranspiration (*PE*) by means of Equations 16 to 24. The code of the first module, which turned out to be the most complex to program, is available in BASIC language, from the author.

An interesting aspect of this first module is establishing the initial soil moisture conditions in the first year of records. This was resolved by trial and error, assigning values to *Ss* and *Su* in January and contrasting them with those of December of that year; when they were equal, the problem was solved. For subsequent years, the December values of a year are used as the initial values for January of the following year. Magnitudes of 25 mm for Ss_{max} and 150 mm for Su_{max} were adopted. Table 3 shows the pairs of values calculated by trial and error for Ss_{inic} and Su_{inic}, in the first year of records, as well as the sets of coefficients of the SWB obtained for each of the three climatological stations processed.

Table 3. Values of the coefficients of the SWB estimated with the historical records for the storage of soil moisture indicated in the three climatological stations processed in the state of San Luis Potosí, Mexico.

Station	Villa de Arriaga	Río Verde	Xilitla
	Su_{max} = 150 mm	Su_{max} = 150 mm	Su_{max} = 150 mm Ss_{inic} = 22.5 mm;Su_{inic} = 150

	Ss_{inic} = 0 mm; Su_{inic} = 0 mm				Ss_{inic} = 0 mm; Su_{inic} = 7.3 mm				mm			
Month	Alfa (α)	Beta (β)	Gamma (γ)	Delta (δ)	Alfa (α)	Beta (β)	Gamma (γ)	Delta (δ)	Alfa (α)	Beta (β)	Gamma (γ)	Delta (δ)
JAN	0.287	0.012	0.161	0.643	0.398	0.011	0.000	0.694	0.994	0.433	0.120	0.115
FEB	0.273	0.004	0.020	0.768	0.310	0.012	0.000	0.808	0.984	0.313	0.128	0.154
MAR	0.199	0.006	0.000	0.871	0.210	0.000	0.000	0.857	0.958	0.157	0.094	0.318
APR	0.197	0.001	0.000	0.825	0.289	0.023	0.000	0.678	0.936	0.245	0.168	0.264
MAY	0.318	0.007	0.000	0.530	0.292	0.001	0.000	0.693	0.925	0.300	0.378	0.149
JUN	0.448	0.061	0.108	0.495	0.558	0.075	0.000	0.472	0.990	0.712	1.594	0.018
JUL	0.609	0.082	0.386	0.444	0.546	0.116	0.138	0.511	0.989	0.822	1.791	0.018
AUG	0.583	0.068	0.080	0.484	0.574	0.050	0.050	0.614	0.999	1.000	1.695	0.014
SEP	0.612	0.095	0.236	0.387	0.748	0.171	0.096	0.327	1.000	1.000	2.572	0.000
OCT	0.463	0.037	0.000	0.545	0.613	0.035	0.007	0.539	0.998	0.000	1.148	0.037
NOV	0.313	0.000	0.000	0.863	0.447	0.005	0.000	0.750	0.995	0.876	0.253	0.125
DEC	0.271	0.019	0.006	0.858	0.383	0.016	0.000	0.729	0.992	0.419	0.104	0.197

The subprogram SPDI2 incorporates the second module, in which the *moving sums* of duration k, in months, are calculated to form the sequences D_l^k, whose basic statistical parameters (mean, standard deviation and coefficient of asymmetry) and minimum and maximum values are quantified. This module includes the fitting of the LL3 distribution to the formed sequence D_l^k by means of Equations 25 to 32, and ends with the application of Equation 26 to estimate the probabilities of non-exceedance [$P(X \leq x)$] of each element in the sequence. Relevant results of this module are the shape, scale and location parameters of the LL3 function.

Next, the **SPDI3** program is completed with the third module, which calculates the SPDI values by only transforming its probabilities of non-exceedance to standard normal variables (Z), by means of Equations 33 to 36. In addition, two statistical parameters of SPDI = Z are calculated, the mean and its variance, as well as its number of negative elements. These quantities are actually *measurements of goodness of fit*, since their values must be zero, one, and half the number of sequences formed (*ns*). The extreme minimum value of the SPDI is also detected. Finally, in this module the number of light, moderate, severe and extreme droughts is counted, corresponding to the number of SPDI values between 0 and -1.00, from -1.00 to -1.50, from -1.50 to -2.00 and that are less than -2.00, respectively.

Contrasts of the SPDI

Based on the SPDI3 program, the historical precipitation and average monthly temperature records from the three climatological stations in dissimilar climatic locations are processed for the three drought durations (*k*) established. The last three columns in Table 4, Table 5 and Table 6 show the results of the aforementioned program. Columns two, three and four present the results of the SPI index, which come from the calculations made by Campos-Aranda (2017), and columns five, six and seven show the results of the SPEI index, applied using the Thornthwaite method (Equations 16 to 24).

Table 4. Results of meteorological drought indices shown with monthly data from the climatological station Villa de Arriaga in the state of San Luis Potosí, Mexico.

Numerical Concepts	**SPI**			**SPEI**			**SPDI**		
	Durations (months)			**Durations (months)**			**Durations (months)**		
	6	**12**	**24**	**6**	**12**	**24**	**6**	**12**	**24**
Moving sum (No.)	(631)	(625)	(613)	(631)	(625)	(613)	(631)	(625)	(613)
minimum	0.0	30.0	156.7	-629.4	-834.6	-1616.7	-287.4	-377.7	-676.1
maximum	946.5	1456.5	2011.5	463.4	648.9	398.2	727.5	1292.1	1577.6
Arithmetic mean	179.9	360.3	725.9	-231.5	-460.8	-917.1	0.575	3.486	13.879
Standard Dev.	157.4	210.6	356.0	144.4	218.3	371.3	150.0	249.7	423.9
Skew coef.	1.496	1.425	1.181	1.090	1.243	0.974	1.429	1.414	1.221
PDF: GM2 or									

LL3									
Shape param.(λ)	1.3436	3.1500	4.3579	0.164	0.173	0.135	0.224	0.214	0.173
Scale param. (σ)	139.16	114.39	166.58	453.9	646.4	1451.3	323.4	573.1	1239.3
Location param.	-	-	-	-705.6	-1139.4	-2410.9	-350.7	-614.4	-1287.1
Drought Index									
Arithmetic mean	0.0053	-0.0074	-0.0417	-0.0022	-0.0006	-0.0039	-0.0008	0.0008	-0.0028
Variance	0.9767	1.0159	1.2073	1.0044	0.9854	0.9888	1.0089	0.9876	0.9936
Minimum value	-2.760	-2.320	-4.241	-4.118	-2.229	-2.280	-3.198	-2.147	-2.128
No. negative val.	314	319	314	326	323	304	332	337	309
% of droughts	49.8	51.0	51.2	51.7	51.7	49.6	52.6	53.9	50.4
Types Droughts									
% light	33.0	32.5	35.6	36.0	33.8	32.3	37.9	35.8	33.4
% moderate	8.7	11.4	6.5	9.0	11.8	9.3	8.1	11.0	7.3
% severe	7.0	6.1	4.7	4.9	4.8	6.4	4.4	6.9	9.1
% extreme	1.1	1.1	4.4	1.7	1.3	1.6	2.2	0.2	0.5

Table 5. Results of meteorological drought indices shown with monthly data from the Río Verde climatological station in the state of San Luis Potosí, Mexico.

Numerical Concepts:	**SPI**			**SPEI**			**SPDI**		
	Durations (months)			**Durations (months)**			**Durations (months)**		
	6	**12**	**24**	**6**	**12**	**24**	**6**	**12**	**24**
Moving sum (No.)	(643)	(637)	(625)	(643)	(637)	(625)	(643)	(637)	(625)
minimum	4.1	177.4	551.8	-694.0	-988.9	-1754.3	-372.0	-454.8	-675.9
maximum	869.7	1191.9	1922.6	170.5	114.9	-295.0	588.0	853.2	1198.4
Arithmetic mean	263.7	524.9	1049.8	-296.9	-593.9	-1187.8	-0.067	-1.46	-5.31
Standard Dev.	167.9	173.1	268.8	155.0	190.0.	299.3	160.3	253.7	410.4
Skew coef.	0.785	0.604	0.501	0.240	0.463	0.325	0.833	0.628	0.550
PDF: GM2 or LL3									
Shape param. (λ)	2.1266	9.3266	15.6140	0.041	0.084	0.072	0.167	0.147	0.132
Scale param. (σ)	124.01	56.2781	67.2355	2116.3	1265.4	2358.1	503.8	933.0	1707.8

Location param.	-	-	-	-2417.0	-1872.8	-3563.7	-527.0	-967.4	-1761.0.
Drought Index									
Arithmetic mean	0.0160	0.0022	-0.0137	-0.0131	-0.0050	-0.0059	0.0000	- 0.0003	-0.0009
Variance	0.9616	0.9950	1.0531	0.9939	0.9938	1.0008	1.0051	1.0053	1.0042
Minimum value	-2.699	-2.593	-3.167	-2.471	-2.205	-1.963	-3.131	-2.124	-1.863
No. negative val.	299	325	338	325	340	311	341	326	329
% of droughts	46.5	51.0	54.1	50.5	53.4	49.8	53.0	51.2	52.6
Types Droughts									
% light	29.5	35.5	37.1	32.2	35.9	30.9	37.8	33.6	32.6
% moderate	10.9	11.0	10.9	11.2	11.3	13.9	9.5	12.9	16.2
% severe	4.0	2.4	4.2	5.6	5.0	5.0	3.7	4.2	3.8
% extreme	2.0	2.2	1.9	1.6	1.1	0.0	2.0	0.5	0.0

Table 6. Results of the meteorological drought indices shown with monthly data from the Xilitla climatological station in the state of San Luis Potosí, Mexico.

Numerical Concepts:	**SPI**			**SPEI**			**SPDI**		
	Durations (months)			**Durations (months)**			**Durations (months)**		
	6	**12**	**24**	**6**	**12**	**24**	**6**	**12**	**24**
Moving sum (No.)	(595)	(589)	(577)	(595)	(589)	(577)	(595)	(589)	(577)
minimum	195.0	1526.4	3427.1	-278.0	293.7	976.6	- 1172.4	- 1293.0	-2115.2
maximum	3444.7	4423.2	8085.8	2729.6	3274.8	5828.3	1303.5	1633.7	2541.2
Arithmetic mean	1376.3	2740.5	5455.2	795.4	1581.4	3136.3	-0.957	-5.120	35.240
Standard Dev.	750.6	614.3	870.7	667.1	632.6	913.6	413.1	609.9	880.7
Skew coef.	0.470	0.150	0.229	0.581	0.147	0.214	0.205	0.175	0.204
PDF: GM2 or LL3									
Shape param. (λ)	3.0199	19.371	39.248	0.136	0.027	0.047	0.039	0.036	0.047
Scale param. (σ)	455.72	141.47	138.99	2689.1	13300.7	11006.0	5971.0	9658.5	10636.7
Location param.	-	-	-	-	-	-7898.5	-	-	-

				1973.6	11722.0		5980.6	9674.4	10699.8
Drought Index									
Arithmetic mean	-0.011	-0.0178	-0.0055	-0.0007	-0.0203	-0.0113	-0.0137	-0.0145	-0.0106
Variance	1.0476	1.07966	1.0288	1.0017	1.0036	1.0063	0.9987	0.9993	1.0004
Minimum value	-2.530	-3.159	-3.293	-1.847	-1.996	2.321	-2.682	-2.072	-2.314
No. negative val.	287	293	303	305	315	306	304	308	295
% of droughts	48.2	49.7	52.5	51.3	53.5	53.0	51.1	52.3	51.1
Types Droughts									
% light	28.9	30.2	35.2	30.6	32.3	32.6	34.8	31.6	31.9
% moderate	10.9	10.9	10.1	17.5	14.8	13.5	8.4	13.9	12.3
% severe	6.6	5.6	4.9	3.2	6.5	5.7	5.7	6.6	6.4
% extreme	1.8	3.1	2.4	0.0	0.0	1.2	2.2	0.2	0.5

Tables 4, 5 and 6 present four types of numerical values: [1] those associated with the sequences (D_l^k) or moving sums, (2) those related to the fitting of the mixed gamma (GM2) and log-logistic (LL3) distributions, [3] those corresponding to the statistical parameters of the SPI, SPEI and SPDI, which are indicators of the statistical quality of the fitting obtained and, therefore, reflect the accuracy of the results and [4] those associated with the four types of droughts studied.

Discussion

Generalities

As seen in Tables 4, 5 and 6, the statistical parameters of the sequences formed with moving sums are quite different in each index. This is due to the fact that in the SPI they come from the monthly precipitation, in the SPEI from the differences (*d*) between the precipitation minus the monthly potential evapotranspiration, and in the SPDI from Palmer moisture deviations ($\hat{d}$).

Regarding the quality indicators of the fittings obtained, the SPDI was found to result in the best fittings in the three processed climatological stations, since their mean and variance are closer to zero and one. The worst fittings were those resulting from the SPI index. In general, the SPEI and SPDI indices tended to overestimate the number of droughts in the three climatological stations processed, and the SPI's estimates were better for the 6- and 12-month durations.

Finally, regarding the percentages of each type of drought established by the three indices, all report similar values in order of magnitude, but they differ in a specific way for each duration of drought, mainly in relation to severe and extreme droughts.

Based on the results of Tables 4, 5 and 6, it can be deduced that the percentages of the less dispersed and more similar types of drought were obtained with the 12-month duration. Then, taking as a reference the percentages defined by the SPDI, they were compared with those of the SPI and the SPEI, and representative values were selected, adjusting these percentages to a sum of 50%. The Villa de Arriaga resulted in 33%, 11 %, 5% and 1% for light, moderate, severe and extreme droughts, respectively. The Río Verde station resulted in 34%, 11%, 4% and 1% and Xilitla resulted in 31%, 12.5%, 6% and 0.5%, respectively.

Definition of Periods of Drought

Figure 3, Figure 4 and Figure 5 present the graphs of the evolution of the SPDI index for each of the three climatological stations processed, for the duration of 12 months, which was found to be the most convenient for establishing representative values.

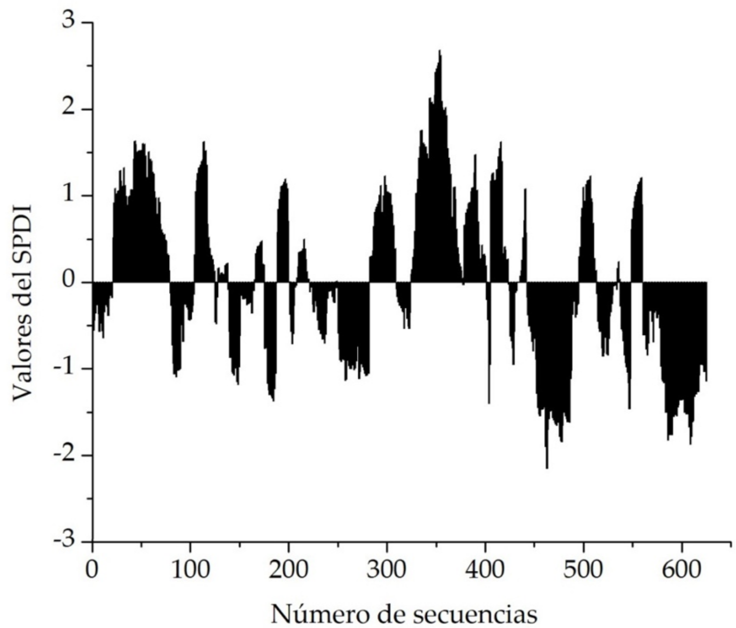

Figure 3. Evolution of the SPDI for 12-month durations in the Villa de Arriaga climatological station in the state of San Luis Potosí, Mexico.

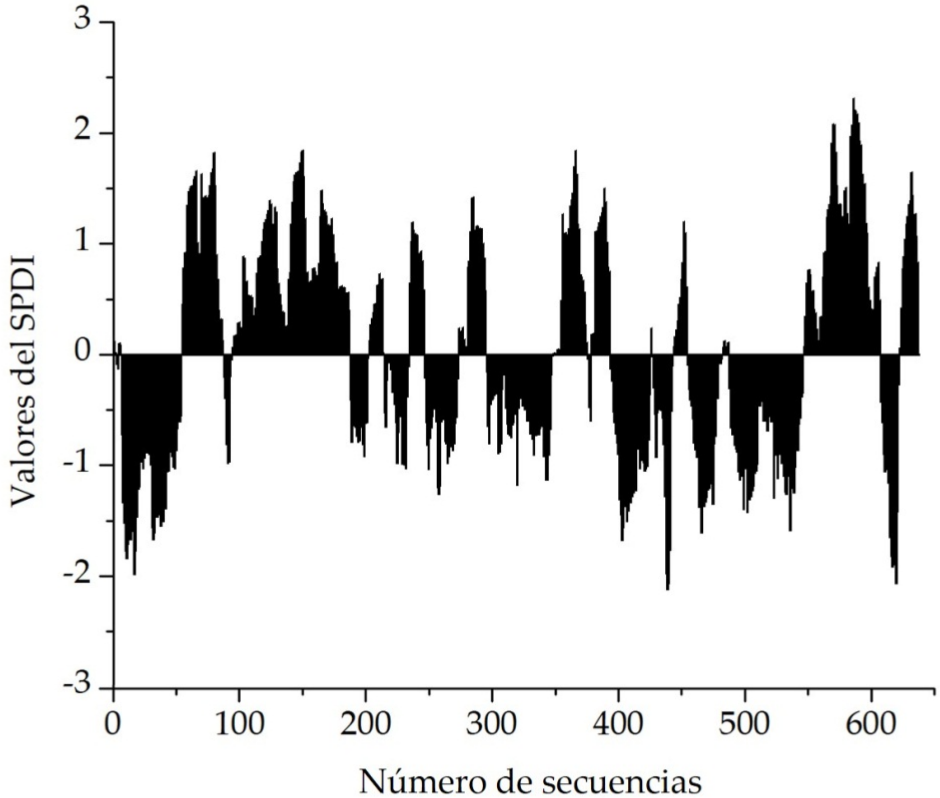

Figure 4. Evolution of the SPDI for 12-month durations in the Río Verde climatological station in the state of San Luis Potosí, Mexico.

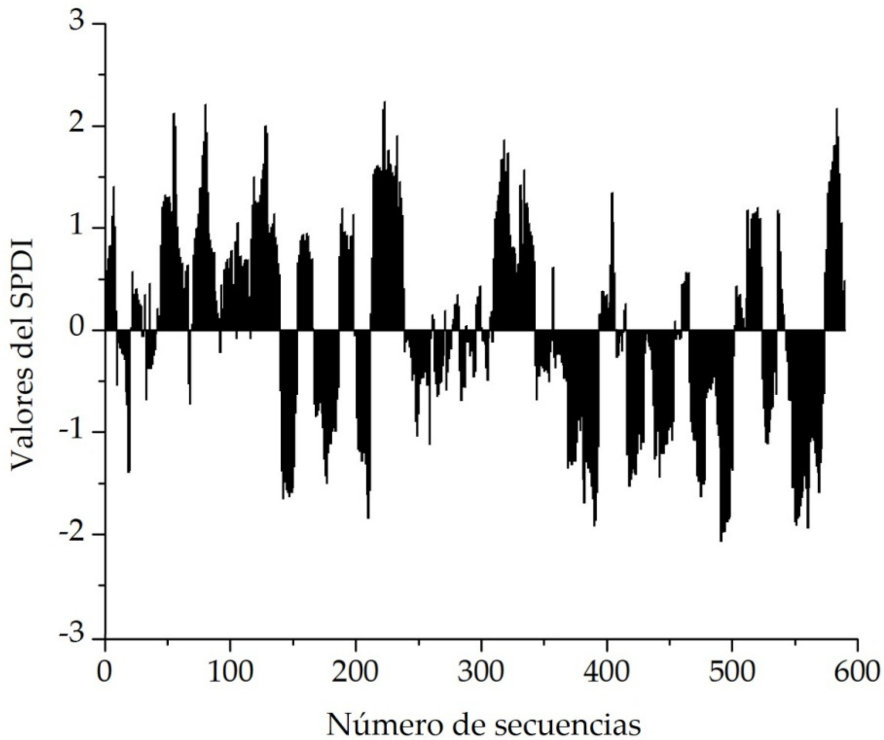

Figure 5. Evolution of the SPDI for 12-month durations in the Xilitla climatological station in the state of San Luis Potosí, Mexico**.**

Figure 3 shows the most severe droughts in duration and magnitude occurred in the last drought period, which contains the extreme minimum value. The second event begins in sequence 443 (October 1999) and ends in 495 (February 2004), with the maximum event occurring in sequence 463 (June 2001). The dates in parentheses are obtained based on Equation 25, clearing NY, whose whole part defines the year by adding it to the beginning of the record, and its fractional portion is defined by multiplying it by 12, resulting in the respective month number. The most critical drought period, the last one, began in sequence 561 (August 2009) and ended in 625 (December 2014).

Figure 4 shows that the first drought was one of the most severe, which began in sequence 7 (June 1962) and ended in 54 (May 1966). The maximum event occurred in sequence 439 (June 1998).

From Figure 5, it can be deduced that the period with the most severe drought was the antepenultimate one, which begins in sequence 466 (September 2004) and ends in 501 (August 2007). The extreme maximum event occurred in sequence 491 (October 2006).

Conclusions

Tables 4, 5 and 6 present the results from comparing the three meteorological drought indices (SPI, SPEI and SPDI) applied in the climatological stations Villa de Arriaga, Rio Verde and Xilitla, located in each of the three dissimilar climates in the state of San Luis Potosí, Mexico: arid, temperate and hot-humid, respectively.

These tables indicate that the statistical parameters of the sequences formed through *moving sums* differed markedly in each index, which is due to its origin: precipitation (P), P differences minus potential evapotranspiration, and moisture deviations ($\hat{d}$).

It is concluded that the SPDI achieves the most reliable results in the three climatological stations processed, since their mean and variance are closer to zero and one. The worst fittings were defined by the SPI index. In general, the SPEI and SPDI indices tend to overestimate the number of droughts in the three climatological stations processed and the SPI estimates are better for the 6- and 12-month durations.

In relation to the percentages of each type of drought established by the three indices, all report similar values in order of magnitude, but they differ in a specific way for each period of drought, mainly in relation to severe and extreme droughts.

The following percentages of light, moderate, severe and extreme meteorological droughts were defined: (1) in Villa de Arriaga 33%, 11%, 5% and 1%; (2) in Río Verde 34%, 11%, 4% and 1%; and (3) in Xilitla 31%, 12.5%, 6% and 0.5%, respectively.

The evolution graphs of the SPDI make it possible to clearly establish the periods of drought, defining their beginning and ending dates as well as their maximum events, both in severity and on the date of occurrence.

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