DOI: 10.24850/j-tyca-14-02-06

Articles

Probabilistic characterization of the dates of occurrence of annual floods using the von Mises distribution

Caracterización probabilística de las fechas de ocurrencia de las crecientes anuales mediante la distribución de von Mises

Daniel Francisco Campos-Aranda[1], ORCID: https://orcid.org/0000-0002-9876-3967

[1]Retired professor at the Autonomous University of San Luis Potosi, Mexico, campos_aranda@hotmail.com

Corresponding author: Daniel Francisco Campos-Aranda, campos_aranda@hotmail.com

Abstract

The planning and management of a river´s water resources, and the preparation of non-structural plans for flood damage mitigation, depend on the relationship between the annual maximum flows and their date of occurrence. Such dates, as they occur all year long, can be treated as *circular data*, whose statistics of mean direction and dispersion or seasonality index define the two parameters of a *von Mises distribution* (dvM). Such distribution allows the probabilistic characterization of the dates of occurrence of annual floods; that is, it defines their probability distribution function. This study describes the dvM and its maximum likelihood parameter estimation method when the annual data or dates are unimodal and cover the entire year. When the annual dates are concentrated in a period of the year, the dvM is fitted with numerical optimization, via the Rosenbrock algorithm. Finally, when dates of occurrence are bimodal, they are represented by a mixture of three dvMs, which are fitted by means of restricted numerical optimization, using the complex algorithm. As a case study, the dates of occurrence of 777 annual floods registered in 21 hydrometric stations of Hydrological Region No. 10 (Sinaloa), Mexico were processed; detailing seven typical applications of the three types of dvM fittings. The conclusions ratify the dvM, as a probabilistic model of the dates of occurrence of annual floods, either unimodal or bimodal.

Keywords: Seasonality indices, von Mises distribution, numerical integration, Rosenbrock algorithm, Complex algorithm, objective functions, mixture of von Mises distributions.

Resumen

La planeación y manejo de los recursos hidráulicos de un río y la elaboración de los planes —de tipo no estructural— de mitigación de daños causados por sus inundaciones dependen de la relación que guarda el gasto máximo anual con su fecha de ocurrencia. Tales fechas, al acontecer durante el año, pueden ser tratadas como *datos circulares*, cuyos estadísticos de dirección media y dispersión, o índice de estacionalidad, definen los dos parámetros de ajuste de la *distribución de von Mises* (dvM), la cual permite la caracterización probabilística de las fechas de ocurrencia de las crecientes anuales; es decir, define su función de distribución de probabilidades. En este estudio se describe la dvM y su método de ajuste por máxima verosimilitud cuando los datos o fechas anuales son unimodales y abarcan todo el año. Cuando las fechas anuales se concentran en una porción del año, la dvM se ajusta con optimización numérica, vía el algoritmo de Rosenbrock. Por último, se describe cómo se representan, con una mezcla de tres dvM, las fechas de ocurrencia que son bimodales, cuyo ajuste, vía optimización numérica restringida, se realizó con el algoritmo Complex. Como un caso de aplicación, se procesaron las fechas de ocurrencia de 777 crecientes anuales registradas en 21 estaciones hidrométricas de la Región Hidrológica No. 10 (Sinaloa), México; se exponen con detalle siete aplicaciones típicas de los tres tipos de ajuste de la dvM. Las conclusiones ratifican a la dvM como modelo probabilístico de las fechas de ocurrencia de las crecientes anuales, sean unimodales o bimodales.

Palabras clave: índices de estacionalidad, distribución de von Mises, integración numérica, algoritmo de Rosenbrock, algoritmo Complex, funciones objetivo, mezcla de distribuciones de von Mises.

Received: 02/08/2021
Accepted: 07/10/2021

Introduction

Generalities

The *Flood Frequency Analysis* (FFA) is perhaps the most important hydrological estimation, as it allows defining the so-called *Design Floods*, which are *predictions* of maximum annual flows associated with low probabilities of being exceeded. Based on the Design Floods, bridges, dikes or retaining walls, rectifications or canalizations and all urban drainage works are dimensioned hydrologically; In addition, they allow the formulation of flood risk and damage mitigation plans (Chen, Guo, Yan, Lui, & Fang, 2010; Khedun, Singh, & Byrd, 2019).

The *univariate* FFA commonly processes the record of maximum annual flows, adopting a probabilistic model or *probability distribution* *function* (PDF), to make the desired inferences or *predictions* (Design Floods). For the FFA results to be reliable, the processed data must be random, several PDFs must be tested and the one that best represents the available registry must be selected (Kite, 1977; Stedinger, Vogel, & Foufoula-Georgiou, 1993; Rao & Hamed, 2000; Meylan, Favre, & Musy, 2012; Stedinger, 2017; Teegavarapu, Salas, & Stedinger, 2019).

On the other hand, the risk of floods and their damages are a direct function of the water volume that runs through the channel and exceeds its limit, overflowing and covering the flood plains. However, the *date of occurrence* of the event is as important as its magnitude and, in some cases, greater, when they occur outside the wet season, since it catches the population off guard causing greater damage (Khedun *et al.*, 2019).

Therefore, knowledge about the probability of flood occurrence throughout the year is vital for the development of non-structural damage mitigation plans, which include preparation for the event, in order to reduce exposure and vulnerability of the population, as well as optimization of the economic resources available for the emergency and accelerating the recovery after the event (Durrans, Eiffe, Thomas, & Goranflo, 2003; Khedun *et al.*, 2019).

Understanding the seasonal behavior of floods is vital in the planning and management of the river's hydraulic resources, both for agricultural and hydroelectric uses, as well as for navigation, recreational uses and other activities associated with bodies of water. For all these reasons, knowing the relationship between the maximum flow and its date of occurrence is extremely important, to the extent that it requires its joint bivariate study (Chen *et al.*, 2010).

The *bivariate* FFA began formally at the beginning of this century (Yue & Rasmussen, 2002) and was generally based on the maximum flow and the volume of the annual floods, using the bivariate Normal distribution and the so-called Logistic model that accepts as *equal marginal* PDFs to the extreme value distributions, the most common being Gumbel and GVE (Escalante-Sandoval & Reyes-Chávez, 2002; Aldama, Ramírez, Aparicio, Mejía-Zermeño, & Ortega-Gil, 2006).

Currently, through the use of the mathematical tool known as "Copulas", bivariate PDFs can be constructed with marginals of different types, because the *copula functions* allow multivariate distributions to be represented from univariate or *marginal* PDFs, regardless of their shape or form type (Kottegoda & Rosso, 2008; Genest & Chebana, 2017; Zhang & Singh, 2006; Zhang & Singh, 2019).

Objective

The *objectives* of this study can be grouped into the following four: (1) To explain I detail the *von Mises Distribution* (dvM, by its acronym in

Spanish) and its maximum likelihood fit method, for data that cover the entire year; (2) to describe in detail the numerical optimization fit technique, via the Rosenbrock algorithm, for data that are concentrated in a period of the year; (3) to chart and represent, with a mixture of three dvM, the dates of occurrence that are bimodal; whose fit, via restricted numerical optimization, was carried out with the Complex algorithm and (4) to process and expose in detail the occurrence dates of 777 annual floods registered in 21 hydrometric stations of the Hydrological Region No. 10 (Sinaloa), Mexico and to explain in detail seven typical applications of the three dvM types of fit.

Operative theory

Directional statistics

When a data is not scalar, but angular or directional, such value can be represented as a *circular data* and having several of them, can be obtained its *directional statistics* that describe them. The theory behind these estimates dates back to the early 1970s and is a simple tool for obtaining similarity measures from the occurrence dates of extreme hydrological events, for example, the floods or maximum flows of a river.

There are various conventions or ways of using the circle to estimate directional statistics (Ramírez-Orozco*,* Gutiérrez-López, & Ruiz-Silva, 2009); in the present study, the convention of Burn (1997) is used, due to its resemblance to Cartesian quadrants. In such a scheme, the advance is counterclockwise, starting on the abscissa axis; therefore, January 1 and December 31 coincide in such beginning (Campos-Aranda, 2017).

The dates of the annual floods occurrence are data that can be treated (drawn and analyzed) as *circular data*, because they are presented within 365 days of each year. Therefore, each date is first converted to a Julian day (1 to 365) and then to radians (0 to 2π). The PDF used to represent such circular data when distributed throughout the year showing a mode is known as the *von Mises distribution* (dvM), which is considered the equivalent of the Normal distribution for scalar data.

Tecnología y Ciencias del Agua
OPEN ACCESS
Check for updates

Seasonality indices

To establish its three values: $\bar{\alpha}$, mean day of floods (*MDF*) and $\bar{r}$, the first step is to convert each date of occurrence of the annual floods to a Julian day (D_i), that is, from 0 to 365; this implies not considering leap years. If a flood occurs on February 29, it is assigned the 28th. The dates for January remain the same, but add 31 to February, 59 to March, 90 to April, and so on until December that 334 is added to them, to obtain the Julian day. Next, the angle α_i in radians corresponding to date *i* of each flood (D_i) is obtained, with the following expression (Burn, 1997; Cunderlik, Ouarda, & Bobée, 2004; Chen, Singh, Guo, Fang, & Liu, 2013; Campos-Aranda, 2017):

$$\alpha_i = 2\pi \frac{D_i}{365} = X_i \quad \text{con } 0 \ \leq \alpha_i \leq 2\pi \tag{1}$$

in which, π is number pi with 3.14159265 as approximate value and X_i is the random variable of the dates of occurrence. Next, the *x* and *y* coordinates of the dates of occurrence of the floods described by the angles α_i are estimated based on the cosines and sines and their mean values are obtained, according to the following equations:

$$\bar{x} = \frac{1}{n}\sum_{i=1}^{n} \cos(\alpha_i) \quad (2)$$

$$\bar{y} = \frac{1}{n}\sum_{i=1}^{n} \text{sen}(\alpha_i) \quad (3)$$

Where, n is the number of occurrence dates from the analyzed annual floods. Now, the mean direction ($\bar{\alpha}$) of the average date of the floods will be:

$$\bar{\alpha} = \text{arc}\tan\left(\frac{\bar{y}}{\bar{x}}\right) \quad (4)$$

The application of the above equation is done by first obtaining the tangent arc of $\bar{y}$ between $\bar{x}$, both with a positive sign called α, in radians; then if $\bar{x}$ and $\bar{y}$ are positive $\bar{\alpha} = \alpha$, if $\bar{x} < 0$ and $\bar{y} > 0$ $\bar{\alpha} = \pi - \alpha$, if both are negative $\bar{\alpha} = \pi + \alpha$ and finally, if $\bar{x} > 0$ and $\bar{y} < 0$ $\bar{\alpha} = 2\pi - \alpha$. The angles α_i and $\bar{\alpha}$ are converted to degrees (0° to 360°) by multiplying them by 57.295755.

The value of $\bar{\alpha}$ in Julian days is called the *mean day of floods* (*MDF*), it is obtained by dividing by 2π and multiplying by 365. The *MDF* index indicates the average date of occurrence of the maximum annual flow in a given basin. Basins with similar *MDF* values can be expected to exhibit similarities in other important hydrological characteristics. Logically, the *MDF* will be related to the size of the basin and its geographical location within the hydrological region studied (Burn, 1997; Cunderlik *et al.*, 2004).

A measure of the variability of the *n* dates of occurrence of the floods, in relation to the *MDF*, can be estimated by calculating the resulting mean, whose expression is:

$$\bar{r} = \sqrt{\bar{x}^2 + \bar{y}^2} \quad (5)$$

The *seasonality index* $\bar{r}$ is a dimensionless measure of the data dispersion, it takes values between zero and one. A unit value indicates that all floods occur on the same date, while a value close to zero implies great variability of occurrences throughout the year.

Ramírez-Orozco *et al.* (2009) establish the following five degrees of seasonality: (1) very strong, when $\bar{r} > 0.90$, (2) strong, when $\bar{r}$ fluctuates between 0.70 and 0.90, (3) medium, when $\bar{r}$ varies from 0.50 to 0.70, (4) low, when $\bar{r}$ changes from 0.10 to 0.50 and (5) very low or weak, when $\bar{r} < 0.10$. Chen *et al.* (2013) indicate that if $\bar{r}$ is close to the unit, it can be expected that a single season or flood season will be dominant.

Tecnología y Ciencias del Agua
OPEN ACCESS
Check for updates

The von Mises distribution

This probabilistic model is commonly used to represent random variables that have direction in two dimensions and a single mode. For this reason, the *von Mises distribution* (dvM) is considered a natural analogy of the Normal model for seasonal data. Its probability density function is the following equation (Metcalfe, 1997; Carta, Bueno, & Ramírez, 2008; Chen *et al.*, 2010):

$$f(x) = \frac{\exp[\kappa \cdot \cos(x-\mu)]}{2\pi I_0(\kappa)} \text{ con } 0 \leq x \leq 2\pi, 0 < \mu \leq 2\pi, \kappa > 0 \tag{6}$$

The dvM is symmetric with its mode at $x = \mu$, which is also its *mean direction* ($\bar{\alpha}$), and the scatter is given by the *concentration parameter* κ (kappa). The denominator of Equation (6) makes the area under the curve unitary and for this reason it is called the *normalization factor* (*FN*); includes the modified Bessel function of the first kind of order zero [$I_0(\kappa)$]. Metcalfe (1997) exposes the results of the maximum likelihood method to estimate the two adjustment parameters of the dvM, using the expressions:

$$\mu = \bar{\alpha} \tag{7}$$

$$\bar{r} = \frac{I_1(\kappa)}{I_0(\kappa)} \tag{8}$$

Equation (7) indicates that μ is calculated with Equation (4). In Equation (8) the numerator is the modified Bessel function of the first kind of order one. The cited Bessel functions of order v can be estimated with the following ascending series (Olver, 1972):

$$I_\nu(z) = \left(\frac{1}{2}z\right)^\nu \sum_{k=0}^{\infty} \frac{\left(\frac{1}{4}z^2\right)^k}{k!\Gamma(\nu+k+1)} \tag{9}$$

Metcalfe (1997) also presents the following three approaches to estimate the concentration parameter κ, according to the value of the reciprocal of $\bar{r}$ (Equation (8)), these are:

$$\frac{1}{\bar{r}} = \frac{I_0(\kappa)}{I_1(\kappa)} = 2\kappa + \kappa^3 + \frac{5\kappa^5}{6} \quad \text{for } \kappa < 0.53 \tag{10}$$

$$\frac{1}{\bar{r}} = \frac{I_0(\kappa)}{I_1(\kappa)} = -0.4 + 1.39\kappa + \frac{0.43}{1-\kappa} \quad \text{for } 0.53 \leq \kappa < 0.85 \tag{11}$$

$$\frac{1}{\bar{r}} = \frac{I_0(\kappa)}{I_1(\kappa)} = \frac{1}{\kappa^3 - 4\kappa^2 + 3\kappa} \quad \text{for } \kappa \geq 0.85 \tag{12}$$

When the seasonality index $\bar{r}$ (Equation (5)) is greater than 0.804 and indicates a strong concentration, Equation (10) is used and when y is less than 0.274 and indicates a low concentration, Equation (12) is applied. For intermediate cases, Equation (11). To estimate the probability of not exceeding a value x, Equation (6) is numerically integrated, that is:

$$F(x)=\frac{1}{2\pi I_0(\kappa)}\int_0^x \exp[\kappa\cdot\cos(x-\mu)] \quad (13)$$

The above expression defines the PDF of the dvM. $I_0(\kappa)$ is estimated with the following ascending series, which comes from Equation (9) (Olver, 1972):

$$I_0(\kappa)=1+\frac{(\kappa^2/4)}{1}+\frac{(\kappa^2/4)^2}{4}+\frac{(\kappa^2/4)^3}{36}+\frac{(\kappa^2/4)^4}{576}+\frac{(\kappa^2/4)^5}{14400}+\cdots \quad (14)$$

Numerical integration of the dvM

To carry out such numerical integration, the Gauss-Legendre quadrature method was adopted, whose univariate operating equation is (Nieves & Domínguez, 1998; Campos-Aranda, 2003):

$$\int_a^b f(x)\,dx \cong \frac{b-a}{2}\sum_{i=1}^{np} w_i \cdot f\left[\frac{(b-a)h_i+b+a}{2}\right] \tag{15}$$

in which, w_i are the coefficients of the method whose abscissas are h_i and np the number of pairs in which the function $f(x)$ is evaluated, with the argument indicated in $f(\cdot)$ of Equation (15). Nieves and Domínguez (1998) expose the coefficients w_i and the abscissas h_i from np = 2 to 6 with 10 digits and Campos-Aranda (2003) those from np = 8 with 9 digits. In Davis and Polonsky (1972) the 12 used pairs of w_i and h_i with 15 digits were obtained, because the *Basic* language accepts 16 digits as double precision variables.

Fitting of the dvM type 1(Standard)

When the annual floods occurrence dates range from January to December, with a monthly histogram that defines a low prevalence during the months of July to October; the application of Equation (13) allows estimating the theoretical probabilities [$F_T(x)$] to contrast them against the empirical ones defined with the Gringorten formula (Chen *et al.*, 2010), which goes as follows:

$$F_E(x) = \frac{m-0.44}{n+0.12} \quad (16)$$

in which, *m* is the order number of the data or date of occurrence in radians ($x = \alpha_i$), when they are located in progressive magnitude and *n* is the total number of data. The fit parameters of the dvM, μ and κ are estimated with equations (7) and (8).

Fitting of the dvM type 2 (local)

When the annual floods occurrence dates cover, for the most part, a fairly defined period of months, for example, from June to October, then the application of Equation (13) is carried out via numerical optimization, to find the values of μ and κ that reduce the sum of the differences between theoretical and empirical probabilities squared (SDTP), that is:

$$\text{Min FO} = \sum_{i=1}^{n}[F_T(x) - F_E(x)]^2 \quad (17)$$

The minimization of the previous objective function is carried out by means of the Rosenbrock algorithm. Logically, previously eliminating the occurrence dates that occurred outside the main or *local* period of the occurrences, to improve the adjustment and therefore, the definition of the dvM.

Fitting of the dvM type 3 (mixed)

When the dates of occurrence of annual floods clearly define two periods with abundance or predominance of occurrences, late fall, and winter (November to March) and summer to early fall (July to October), there are two modes and therefore, the dvM (Equation (13)) is not applicable. However, it is possible to seek to represent the empirical probabilities with a mixture of dvM, each with a weighting factor w_j, seeking to minimize the sum of the differences between the observed probabilities (P_k) and those estimated with the mixture of j dvM (Chen *et al.*, 2010; Carta *et al.*, 2008), that is:

$$\text{Min FO} = \sum_{k=1}^{12}\left\{P_k - \sum_{j=1}^{3}\frac{w_j}{2\pi I_0(\kappa_j)}\int_0^{x_k}\exp\left[\kappa_j\cos\left(x_k-\mu_j\right)\right]dx\right\}^2 \qquad (18)$$

subject to the following restriction:

$$\sum_{j=1}^{3} w_j = 1.0 \quad (19)$$

and complying with: $0 \leq \mu_j \leq 2\pi$, $\kappa_j \geq 0$ and $0 \leq w_j \leq 1$. Equation (18) is written with the approach suggested in this study, for the adjustment of type 3 (mixed), with *k* = 12 intervals to estimate the observed probabilities, that is, by months and a mixture of three dvM, for which, *j* = 3, as suggested by Chen *et al.* (2010).

In relation to the latter, Carta *et al.* (2008) take the *j* value up to nine but indicate that the quality of the fit does not improve beyond six. The approach to find a solution to Equation (18) and estimate the nine fit parameters (w_1, μ_1, κ_1, w_2, μ_2, κ_2, w_3, μ_3, κ_3), was established by means of the Complex algorithm. In this fitting, as in type 1, no data is deleted.

Rosebrook's algorithm

It is a numerical direct search procedure that attempts to define the *minimum* of a nonlinear function of multiple unbounded or restricted random variables. Its mathematical approach is as follows:

$$\text{Min} \quad F(z_1, z_2, \cdots, z_s) \quad (20)$$

Rosebrook's algorithm assumes that the function is unimodal and begins by defining a straight line or search direction, from a given initial point. It then evaluates the *objective function* (FO) at various points on the line and determines the optimum; when this has happened, a new search direction is selected and the process is repeated recursively in stages. In this algorithm it is convenient to give different initial points to search for the global minimum, from the estimated local minima. A more detailed description of the process can be found in Rosenbrock (1960), Kuester and Mize (1973), and Campos-Aranda (2003).

The original computer program in Fortran from the second cited reference, was transformed into Basic language and is called ROSEN, whose main variables and parameters (Campos-Aranda, 2003) are: (1) the number of random variables to optimize (NVA); (2) their initial values; (3) the FO designated FX; (4) the maximum number of evaluations of the objective function (MF), of stages (ME), and of successive failures found in all directions (MC), and (5) the acceptable error (EY) in the FO or difference between the current value and that of the previous stage.

OPEN ACCESS

Check for updates

Complex Algoritm

It is a local exploration numerical technique that is guided by what it finds in its path, it allows minimizing a function of multiple bounded or restricted continuous random variables (z). His mathematical approach is as follows (Box, 1965):

$$\text{Min} \quad \mathrm{F}(z_1, z_2, \cdots, z_s) \tag{21}$$

Subject to *m* dependent variables (y), function of the decision variables (z):

$$\begin{array}{l} y_1 = \mathrm{F}(z_1, z_2, \cdots, z_s) \\ \quad \vdots \qquad\qquad \vdots \\ y_m = \mathrm{F}(z_1, z_2, \cdots, z_s) \end{array} \tag{22}$$

Both variables have lower and upper bounds of type ≤, that is: $z_{inf} \leq z_i \leq z_{sup}$ and $y_{inf} \leq y_j \leq y_{sup}$. For the *Complex algorithm*, its background, a brief description of its operating process and its OPTIM code in Basic can be found in Campos-Aranda (2003). Bunday (1985) has another description and code for this search method.

The main designations in the OPTIM code are NX and NY, which define the number of decision and dependent variables. MI is the maximum number of evaluations of the objective function and NQ the number of such calculations between printing of results. These variables are defined in the data reading subroutine.

An important advantage of the OPTIM code is that it allows easy access to the limits (L = lower, U = upper), names and initial values of the variables, in the cited subroutine, by means of the following designations: XL(I), XU(I), XN(I), X(I), YL(J), YU(J), YN(J), and Y(J); where I is the number of decision variables and J is the number of constraints. The Complex algorithm works with two convergence criteria FA and FR, for the absolute and relative deviations of the *objective function* (FO), which is defined at the end of the program.

Data processed

In this study, the dates of occurrence of 777 annual floods processed by Campos-Aranda (2014), in 21 hydrometric stations of Hydrological Region No. 10 (Sinaloa), Mexico, were used. Due to the above, Table 1 presents a summary of such information, which includes: names of the gauging stations, basin areas, recording periods and two seasonal indices of their floods occurrence dates.

Table 1. General data and seasonal indices of the annual floods registered in the 21 hydrometric stations of Hydrological Region No. 10 (Sinaloa), Mexico.

No.	Name	*A*	Record (years)	*MDF*	Date	$\bar{r}$
1	Huites	26 057	1942-1992 (51)	293.6	Oct. 21	0.3739
2	San Francisco	17 531	1941-1973 (33)	285.6	Oct. 13	0.4133
3	Santa Cruz	8 919	1944-2002 (52)	283.7	Oct. 11	0.5507
4	Jaina	8 179	1942-1998 (56)	276.9	Oct. 4	0.5281
5	Palo Dulce	6 439	1958-1986 (21)	292.8	Oct. 20	0.3405
6	Ixpalino	6 166	1953-1999 (45)	279.7	Oct. 7	0.6214
7	La Huerta	6 149	1970-1999 (28)	312.3	Nov. 8	0.4219
8	Chinipas	5 098	1965-2002 (24)	269.9	Sep. 27	0.4457
9	Tamazula	2 241	1963-1999 (32)	252.9	Sep. 10	0.5928
10	Naranjo	2 064	1939-1984 (45)	243.6	Sep. 1	0.7047
11	Acatitán	1 884	1955-2002 (43)	256.3	Sep. 13	0.7497
12	Guamúchil	1 645	1940-1971 (32)	238.4	Ago. 26	0.7299
13	Choix	1 403	1956-2002 (38)	243.8	Sep. 1	0.6889
14	Badiraguato	1 018	1974-1999 (26)	263.0	Sep. 20	0.5989
15	El Quelite	835	1961-2001 (33)	252.4	Sep. 09	0.8035
16	Zopilote	666	1939-2001 (56)	239.7	Ago. 28	0.8322
17	Chico Ruiz	391	1977-2002 (19)	237.5	Ago. 25	0.8161
18	El Bledal	371	1938-1994 (56)	238.0	Ago. 26	0.8458
19	Pericos	270	1961-1992 (30)	230.5	Ago. 19	0.8158
20	La Tina	254	1960-1983 (24)	246.0	Sep. 3	0.7207
21	Bamícori	223	1951-1983 (33)	230.8	Ago. 19	0.8810

Acronyms:

A = watershed area, in km^2.

MDF = mean day of floods.

$\bar{r}$ = seasonality index, dimensionless.

On the other hand, Campos-Aranda (2017) shows the annual data of the floods occurrence dates registered in the Guamúchil and Huites gauging stations, with 32 and 51 years of recording. These series will no longer change, since they cover the start of their operation to the closure of such stations due to the construction of the Eustaquio Buelna and Luis Donaldo Colosio reservoirs. These records were processed to show two types of fit of the dvM.

In addition, in the Campos-Aranda (2014) reference, the cited annual data from the San Francisco and Bamicori hydrometric stations are exposed, both with 33 years of registration, which were also processed to illustrate two types of fit of the dvM. Finally, some of the annual data series analyzed are exposed, which illustrate important results; Such is the case of the Palo Dulce, La Huerta and Jaina stations, to mention a few examples.

Results and their discussion

Fitting of the dvM to the dates of occurrence

Table 2 shows the 21 results of the dvM fit, by means of equations (7) and (8), to all the data of each record. It can be observed in the last column that the minimum values of the SDPC (Equation (17)), occur in the stations of Palo Dulce, La Huerta, Chinipas and Tamazula. Only these four hydrometric stations define or accept the type 1 fit of the dvM.

OPEN ACCESS

Check for updates

Table 2. Fit parameters of the von Mises distribution for the dates of occurrence of the annual floods registered in the 21 hydrometric stations of Hydrological Region No. 10 (Sinaloa), Mexico.

No.	Name	*n*	*μ*	*κ*	*FN*	SDPC
1	Huites	51	5.054255	0.783421	7.2849	0.321
2	San Francisco	33	4.916883	0.756742	7.2154	0.195
3	Santa Cruz	52	4.883268	0.666516	7.0006	0.269
4	Jaina	56	4.766349	0.680831	7.0327	0.286
5	Palo Dulce	21	5.040438	0.806016	7.3459	0.041
6	Ixpalino	45	4.816023	0.623589	6.9090	0.623
7	La Huerta	28	5.374923	0.750965	7.2008	0.059
8	Chinipas	24	4.645802	0.734899	7.1606	0.110
9	Tamazula	32	4.353449	0.640634	6.9446	0.324
10	Naranjo	45	4.193736	0.577105	6.8173	0.809
11	Acatitán	43	4.411395	0.553888	6.7744	1.113
12	Guamúchil	32	4.103816	0.563965	6.7928	0.639
13	Choix	38	4.197106	0.585610	6.8335	0.726
14	Badiraguato	26	4.527146	0.636949	6.9368	0.265
15	El Quelite	33	4.344669	0.527901	6.7286	0.999
16	Zopilote	56	4.126714	0.516567	6.7094	1.870
17	Chico Ruiz	19	4.087486	0.524164	6.7222	0.547
18	El Bledal	56	4.097247	0.510294	6.6989	1.891
19	Pericos	30	3.967493	0.524331	6.7225	0.851
20	La Tina	24	4.233593	0.568730	6.8016	0.524
21	Bamícori	33	3.972656	0.494675	6.6735	1.219

Acronyms:

n = data number.

μ = mean direction, in radians.

k = concentration parameter, dimensionless.

FN = normalization factor, in radians.

SDPC = sum of differences between theoretical probabilities and empirical squared (Equation (17)), dimensionless.

Table 3 and Table 4 show the processed data and the results of the Palo Dulce and La Huerta stations. Figure 1 and Figure 2 show the monthly histograms of occurrence dates and their PDF curve (Equation (13)).

Tecnología y Ciencias del Agua
OPEN ACCESS
Check for updates

Table 3. Annual floods flows, dates of occurrence and their theoretical and empirical probabilities of non-exceedance in Palo Dulce hydrometric station of the Hydrological Region No. 10 (Sinaloa), Mexico.

No.	Q_{max} (m^3/s)	Month	Day	Julian day	α_i ordered (radians)	$F_T(x)$	$F_E(x)$
1	455	Mar	6	65	0.223785	0.0362	0.0265
2	743	Dec	9	343	0.430355	0.0644	0.0739
3	6800	Jan	13	13	0.912353	0.1149	0.1212
4	347	Sep	11	254	1.118923	0.1317	0.1686
5	584	Dec	11	345	2.392775	0.2142	0.2159
6	481	Aug	1	213	3.408413	0.3072	0.2633
7	1360	Dec	23	357	3.614984	0.3363	0.3106
8	674	Aug	22	234	3.666626	0.3444	0.3580
9	635	Dec	15	349	3.683840	0.3472	0.4053
10	530	Aug	2	214	3.873196	0.3801	0.4527
11	1100	Jul	17	198	4.028124	0.4107	0.5000
12	390	Aug	13	225	4.372409	0.4909	0.5473
13	1283	Oct	29	302	4.699478	0.5809	0.5947
14	688	Oct	30	303	4.716693	0.5859	0.6420
15	1370	Feb	22	53	5.198691	0.7313	0.6894
16	2245	Nov	9	313	5.215905	0.7365	0.7367
17	383	Jul	29	210	5.388047	0.7875	0.7841
18	740	Oct	1	274	5.904473	0.9232	0.8314
19	951	Sep	30	273	5.938902	0.9310	0.8788
20	843	Jan	25	25	6.007758	0.9461	0.9261
21	1112	May	19	139	6.145472	0.9744	0.9735

Table 4. Flows of annual floods, dates of occurrence and their theoretical and empirical probabilities of non-exceedance at the La Huerta hydrometric station of the Hydrological Region No. 10 (Sinaloa), Mexico.

No.	Q_{max} (m^3/s)	Month	Day	Julian day	α_i ordered (radians)	$F_T(x)$	$F_E(x)$
1	659	Jan	5	5	0.086071	0.0185	0.0199
2	530	Oct	26	299	0.172142	0.0360	0.0555
3	1931	Oct	30	303	0.223785	0.0460	0.0910
4	1251	Feb	21	52	0.430355	0.0828	0.1266
5	1260	Dec	25	359	0.895139	0.1476	0.1622
6	273	Aug	31	243	1.084495	0.1683	0.1977
7	1250	Aug	28	240	1.618136	0.2148	0.2333
8	331	Aug	31	243	3.287913	0.3264	0.2688
9	1605	Jan	25	25	3.511698	0.3596	0.3044
10	1475	Oct	8	281	3.597769	0.3695	0.3400
11	609	Dec	10	344	4.131410	0.4475	0.3755
12	1303	Mar	4	63	4.183052	0.4568	0.4111
13	683	Dec	15	349	4.183052	0.4568	0.4467
14	1774	Jan	13	13	4.251909	0.4697	0.4822
15	934	Oct	22	295	4.406837	0.5011	0.5178
16	1003	Dec	25	359	4.424052	0.5048	0.5533
17	840	Jul	10	191	4.510123	0.5238	0.5889
18	1111	Nov	19	323	4.837192	0.6042	0.6245
19	1076	Dec	29	363	5.078191	0.6707	0.6600

No.	Q_{max} (m^3/s)	Month	Day	Julian day	α_i ordered (radians)	$F_T(x)$	$F_E(x)$
20	1663	Dec	11	345	5.147048	0.6904	0.6956
21	1318	Jan	10	10	5.215905	0.7104	0.7312
22	1919	Sep	13	256	5.560189	0.8113	0.7667
23	195	Sep	4	247	5.921687	0.9122	0.8023
24	207	Sep	19	262	5.938902	0.9167	0.8378
25	474	Sep	14	257	6.007758	0.9345	0.8734
26	386	Apr	4	94	6.179900	0.9765	0.9090
27	138	Jul	28	209	6.179900	0.9765	0.9445
28	265	Jul	23	204	6.248758	0.9923	0.9801

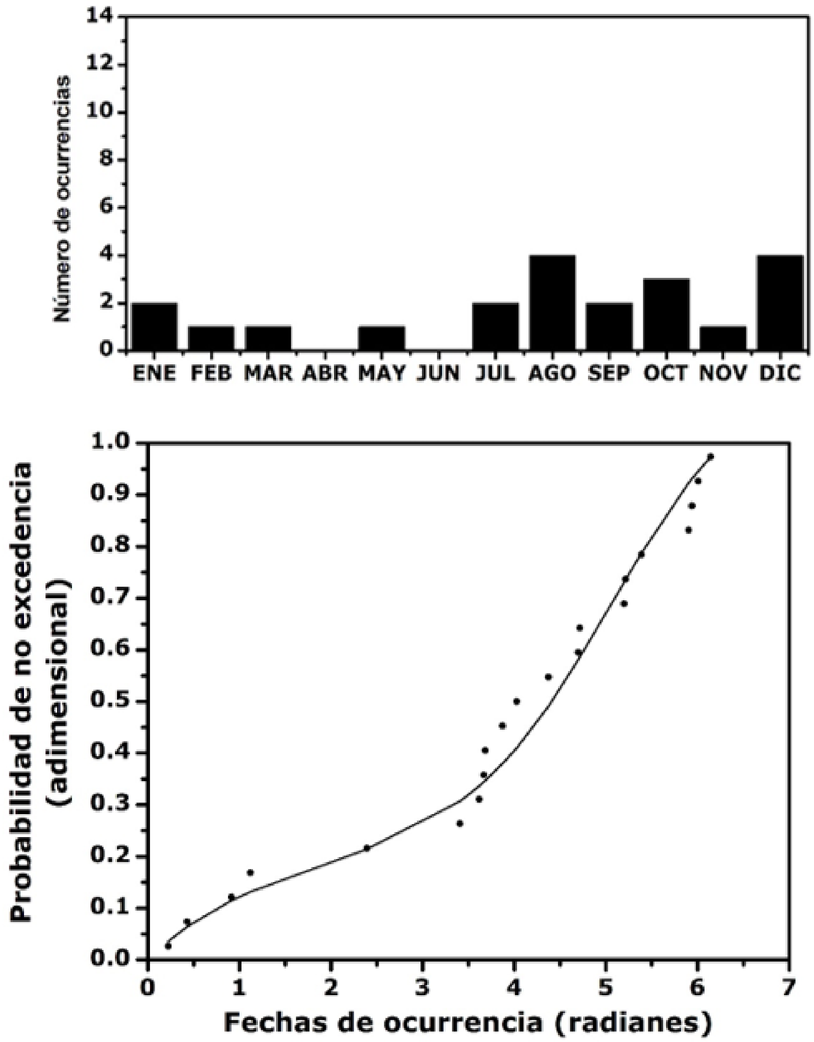

Figure 1. Monthly histogram and type 1 fit of the von Mises distribution to the dates of occurrence of the annual floods recorded at the Palo Dulce hydrometric station. Abscissa exis: Occurrence dates (radians); ordinate axis: non-exceedance probability (adimensional).

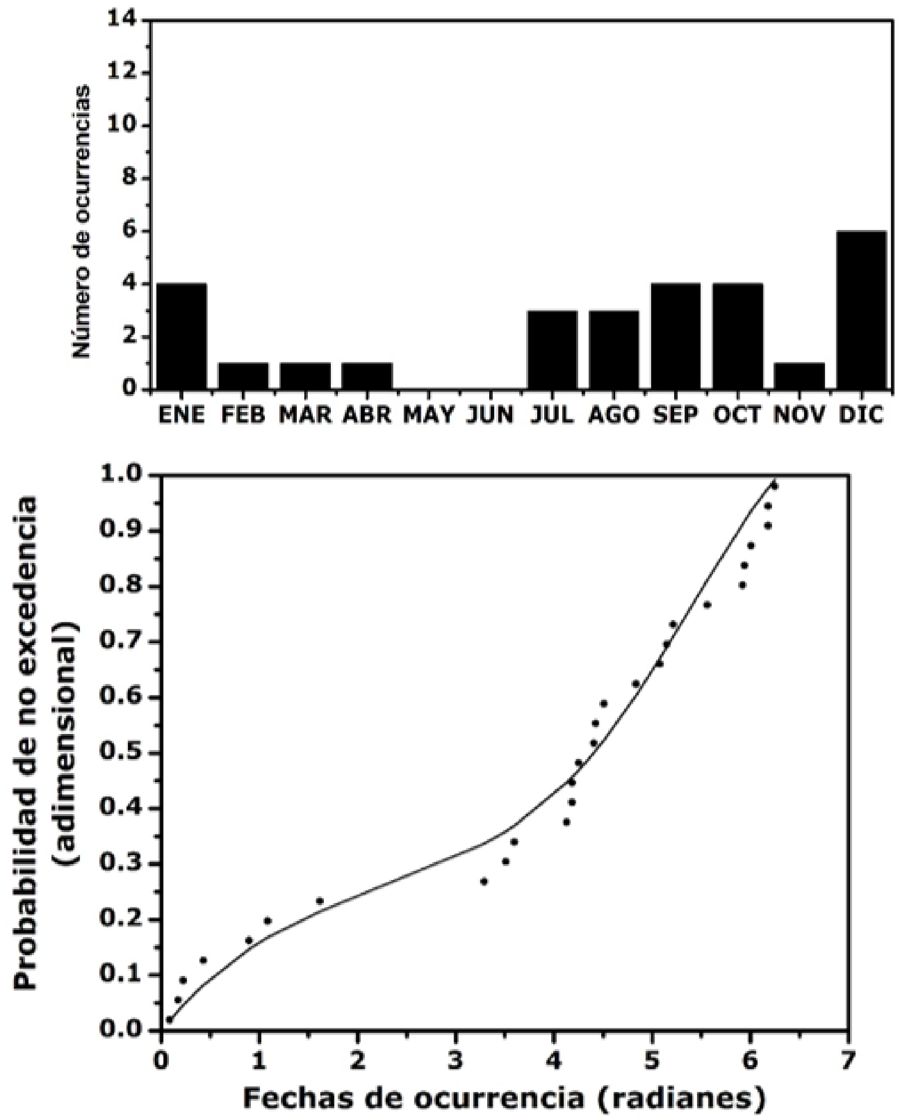

Figure 2. Monthly histogram and type 1 fit of the von Mises distribution to the annual floods occurrence dates recorded at the La Huerta hydrometric station. Abscissa exis: Occurrence dates (radians); ordinate axis: non-exceedance probability (adimensional).

Occurrence Dates by months and dvM type 2 fittings

Table 5 details the number of the annual floods occurrence dates in the 12 months of the year, at the 21 processed hydrometric stations. The three stations that were adopted to illustrate type 2 fit of the dvM, that is, those of local amplitude, are shaded, which are: (1) Bamicori with a record of 33 dates from June to October; (2) El Bledal with a record of 55 dates from July to November, after eliminating the data for January and (3) Guamúchil, with a record of 29 dates from June to October, after discarding two data in January and one in February. For illustrative purposes, only the type 2 fits of the Bamícori and Guamúchil stations are detailed.

Tecnología y Ciencias del Agua

OPEN ACCESS

Check for updates

Table 5. Number of monthly occurrences of the annual floods registered in the 21 hydrometric stations of the Hydrological Region No. 10 (Sinaloa), Mexico.

No.	Name:	J	F	M	A	M'	J	J'	A'	S	O	N	D
1	Huites	8	4	1	0	0	0	6	10	6	7	1	8
2	San Francisco	5	2	0	0	0	0	4	9	3	3	1	6
3	Santa Cruz	11	0	0	0	0	0	4	10	11	9	3	4
4	Jaina	6	3	0	0	0	1	3	15	10	9	2	7
5	Palo Dulce	2	1	1	0	1	0	2	4	2	3	1	4
6	Ixpalino	4	0	0	1	0	0	2	10	9	11	1	7
7	La Huerta	4	1	1	1	0	0	3	3	4	4	1	6
8	Chinipas	1	2	0	0	0	0	5	5	1	5	1	4
9	Tamazula	1	0	1	1	0	0	6	5	9	6	0	3
10	Naranjo	3	0	1	0	0	1	5	16	10	8	0	1
11	Acatitán	1	0	1	0	0	0	5	13	10	11	0	2
12	Guamúchil	2	1	0	0	0	1	5	12	8	3	0	0
13	Choix	2	0	1	0	0	0	7	12	9	4	1	2
14	Badiraguato	2	1	0	0	0	0	3	5	8	3	2	2
15	El Quelite	1	0	0	0	0	0	5	8	10	8	1	0
16	Zopilote	1	0	1	0	0	1	8	20	18	6	1	0
17	Chico Ruiz	0	0	0	0	0	1	4	6	5	2	1	0
18	El Bledal	1	0	0	0	0	0	12	23	12	7	1	0
19	Pericos	0	1	0	0	0	0	10	10	5	4	0	0
20	La Tina	1	0	0	0	0	0	4	7	7	3	0	2
21	Bamícori	0	0	0	0	0	1	8	13	7	4	0	0

The application of the *Rosenbrock algorithm*, for the dvM type 2 fits, was carried out using the following values: MF = 500, ME = 30, MC = 50 and EY = $1.0 \cdot 10^{-7}$. The initial values of μ and κ used at the Bamicori station were 4.00 and 0.50, which define an initial FO of 1.193. These values are adopted similar to those calculated in Table 2, for such station.

After six stages and 33 evaluations of the FO, the following data was obtained: FO = 0.049, μ = 3.9755, κ = 3.3077, FN = 39.4687 and the results shown in Table 6 and Figure 3. This fit of the dvM is valid in the interval of occurrences from 3.0641 to 5.1987 radians, corresponding to the following dates: June 27 to October 29 (302-178 = 124 days).

Table 6. Occurrence dates (x in radians) and theoretical and empirical probabilities of non-exceedance for the occurrence dates of the annual floods from the Bamicori hydrometric station.

No.	$x = \alpha_i$	$F_T(x)$	$F_E(x)$	No.	$x = \alpha_i$	$F_T(x)$	$F_E(x)$
1	3.0641	0.066	0.017	18	3.9593	0.490	0.530
2	3.2363	0.107	0.047	19	3.9593	0.490	0.560
3	3.2707	0.118	0.077	20	3.9937	0.514	0.591
4	3.3223	0.135	0.107	21	4.0109	0.526	0.621
5	3.4773	0.198	0.138	22	4.1658	0.630	0.651
6	3.5117	0.214	0.168	23	4.2347	0.674	0.681
7	3.5806	0.250	0.198	24	4.2691	0.695	0.711
8	3.5806	0.250	0.228	25	4.3036	0.716	0.742
9	3.5978	0.259	0.258	26	4.3724	0.754	0.772
10	3.6666	0.298	0.289	27	4.3896	0.763	0.802
11	3.7871	0.373	0.319	28	4.4413	0.789	0.832
12	3.8043	0.385	0.349	29	4.6995	0.891	0.862
13	3.8043	0.385	0.379	30	4.7167	0.896	0.893
14	3.8043	0.385	0.409	31	4.7855	0.914	0.923
15	3.8043	0.385	0.440	32	4.8372	0.926	0.953
16	3.8732	0.431	0.470	33	5.1987	0.977	0.983
17	3.8732	0.431	0.500	-	-	-	-

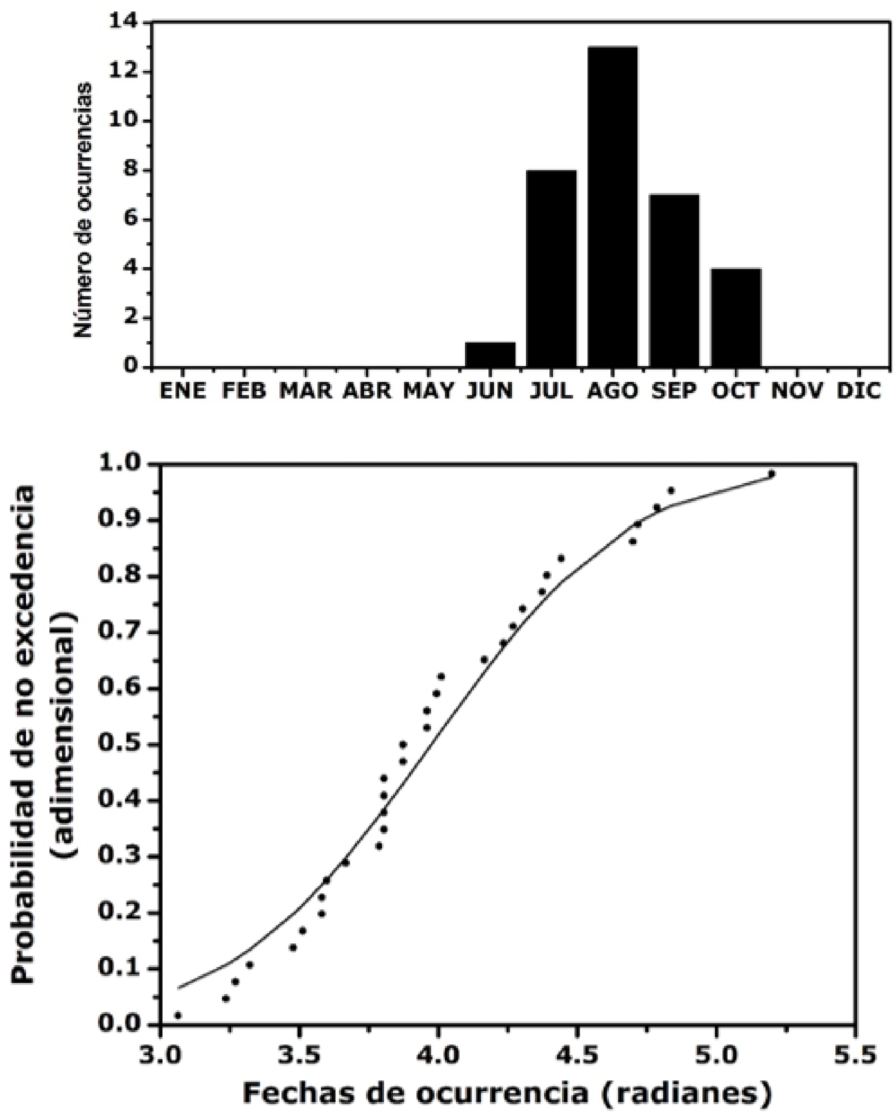

Figure 3. Monthly histogram and type 2 fit of the von Mises distribution to the occurrence dates of the annual floods recorded at the Bamicori hydrometric station. Abscissa exis: Occurrence dates (radians); ordinate axis: non-exceedance probability (adimensional).

On the other hand, the initial values of μ and κ assigned at the Guamúchil station were 4.25 and 0.50, which define an initial FO of 1.040. As already indicated, these values are adopted similar to those calculated in Table 2, for such station.

After 15 stages and 83 evaluations of the FO, the following was obtained: FO = 0.046, μ = 4.0410, κ = 3.7923, FN = 59.3770 and the results shown in Table 7 and Figure 4. This fit of the dvM is valid in the interval of occurrences from 3.0641 to 4.8372 radians, corresponding to the following dates: June 27 to October 8 (281-178 = 103 days).

Table 7. Dates of occurrence (*x* in radians) and theoretical and empirical probabilities of non-exceedance for the dates of occurrence of the annual floods from the Guamúchil hydrometric station.

No.	$x = \alpha_i$	$F_T(x)$	$F_E(x)$	No.	$x = \alpha_i$	$F_T(x)$	$F_E(x)$
1	3.0641	0.041	0.019	16	4.0281	0.491	0.534
2	3.2879	0.086	0.054	17	4.0970	0.543	0.569
3	3.5117	0.165	0.088	18	4.1658	0.593	0.603
4	3.5633	0.189	0.122	19	4.3036	0.689	0.637
5	3.5806	0.198	0.157	20	4.3380	0.711	0.672
6	3.5806	0.198	0.191	21	4.4413	0.772	0.706
7	3.6666	0.244	0.225	22	4.4757	0.791	0.740
8	3.6838	0.254	0.260	23	4.5273	0.817	0.775
9	3.7183	0.275	0.294	24	4.5618	0.833	0.809
10	3.7355	0.285	0.328	25	4.5790	0.841	0.843
11	3.8216	0.342	0.363	26	4.6306	0.862	0.878
12	3.8560	0.366	0.397	27	4.7339	0.898	0.912
13	3.9076	0.402	0.431	28	4.7683	0.909	0.946
14	3.9248	0.415	0.466	29	4.8372	0.927	0.981
15	3.9593	0.440	0.500	-	-	-	-

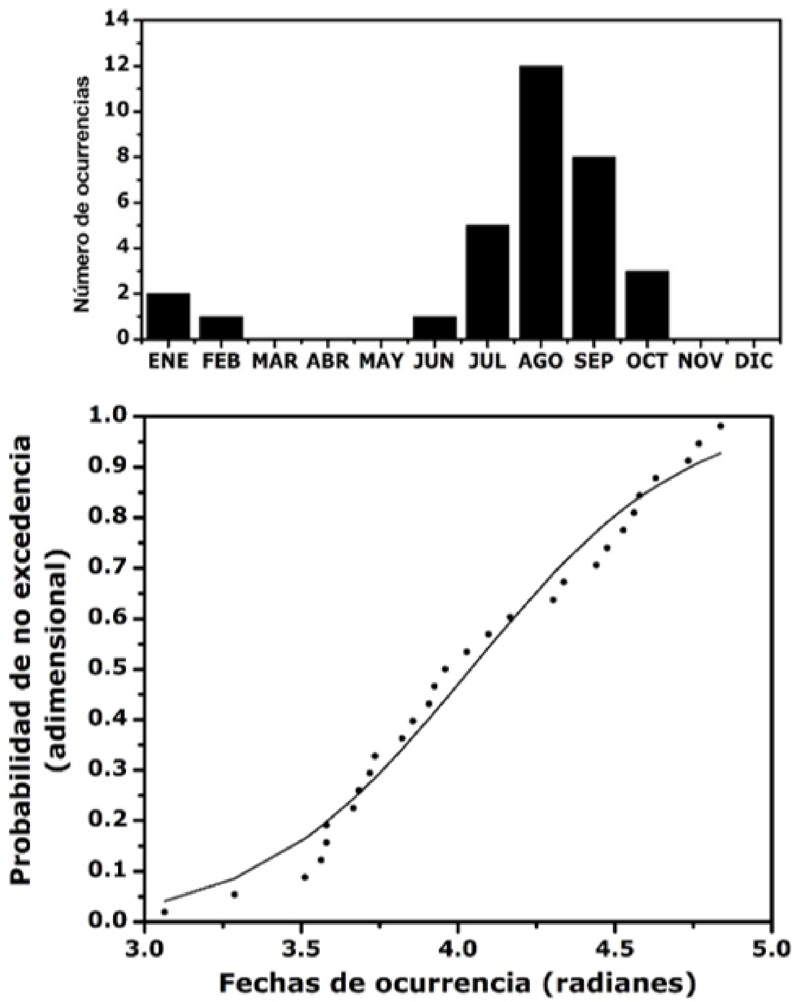

Figure 4. Monthly histogram and type 2 fit of the von Mises distribution to the annual floods occurrence dates recorded at the Guamúchil hydrometric station. Abscissa exis: Occurrence dates (radians); ordinate axis: non-exceedance probability (adimensional).

Tecnología y Ciencias del Agua

OPEN ACCESS

Check for updates

Bimodal dates of occurrence and dvM type 3 fitting

For the application of Equation (18), the upper limits x_k of its integral must first be defined. Since 12 uniform intervals were adopted, corresponding to the months of the year, then each interval limit advances 30° and the 12 cover the circumference of 360°. Then, the limits x_k in radians are obtained by multiplying 0.0174533 by the angles of 30, 60, 90° and so on up to 330 and 360°; to define x_1 = 0.523599, x_2 = 1.047198, x_3 = 1.570797, ..., x_{11} = 5.759589 and x_{12} = 6.283188. The values of the observed probabilities P_k of Equation (18) are obtained by dividing the 12 data from Table 5 relative to the station or record to be processed, by the total number of data n, cited in Table 2.

The application of the *Complex algorithm*, for the fits of type 3 of the dvM, was carried out using the following values: NX = 9, NY = 1; MI = 5000, NQ = 50, FA = 0.0002 and FR = 0.00001. In addition, all the lower limits of the nine fit parameters were defined at 0.10, the upper limits of w_j at 1.00, those of μ_j at 6.283 and those of κ_j at 50. The triples of initial values w, μ and κ, were the following three: (0.25, 0.50, 1.50), (0.60, 4.10, 3.20) and (0.15, 5.50, 2.50). It is observed that the sum of w_j is the unit (0.25 + 0.60 + 0.15) and that the μ_j establish three mean directions, at the beginning of the year, after its middle and towards the end.

Tecnología y Ciencias del Agua

OPEN ACCESS

Check for updates

For the San Francisco hydrometric station, the adopted and quoted initial values of w_i, μ_i and κ_i, define the following summary of optimal parameters in Table 8, after 2075 evaluations of the objective function (FO). Figure 5 shows the contrast of 33 empirical and theoretical non-exceedance probabilities achieved with the type 3 fit of the dvM; corresponding to the 33 flood dates recorded at the San Francisco station.

Table 8. Results of the Complex algorithm for the type 3 fit at the San Francisco hydrometric station of the Hydrological Region No. 10 (Sinaloa), Mexico.

Parameters	Value	No. month	Acumulated		Increments	
			$F_E(x)$	$F_T(x)$	$F_E(x)$	$F_T(x)$
FO initial	0.03876	1	0.1515	0.1566	0.152	0.157
FO final	0.00245	2	0.2121	0.2029	0.061	0.046
w_1	0.12920	3	0.2121	0.2091	0.000	0.006
μ_1	0.10013	4	0.2121	0.2098	0.000	0.001
κ_1	7.21766	5	0.2121	0.2106	0.000	0.001
w_2	0.51320	6	0.2121	0.2241	0.000	0.013
μ_2	3.94560	7	0.3333	0.3594	0.121	0.115
κ_2	6.39697	8	0.6061	0.5940	0.273	0.255
w_3	0.33760	9	0.6970	0.7244	0.091	0.130
μ_3	6.11277	10	0.7879	0.7570	0.091	0.033
κ_3	3.74811	11	0.8182	0.8346	0.030	0.078
Suma de w_j	0.98000	12	1.0000	0.9955	0.182	0.161

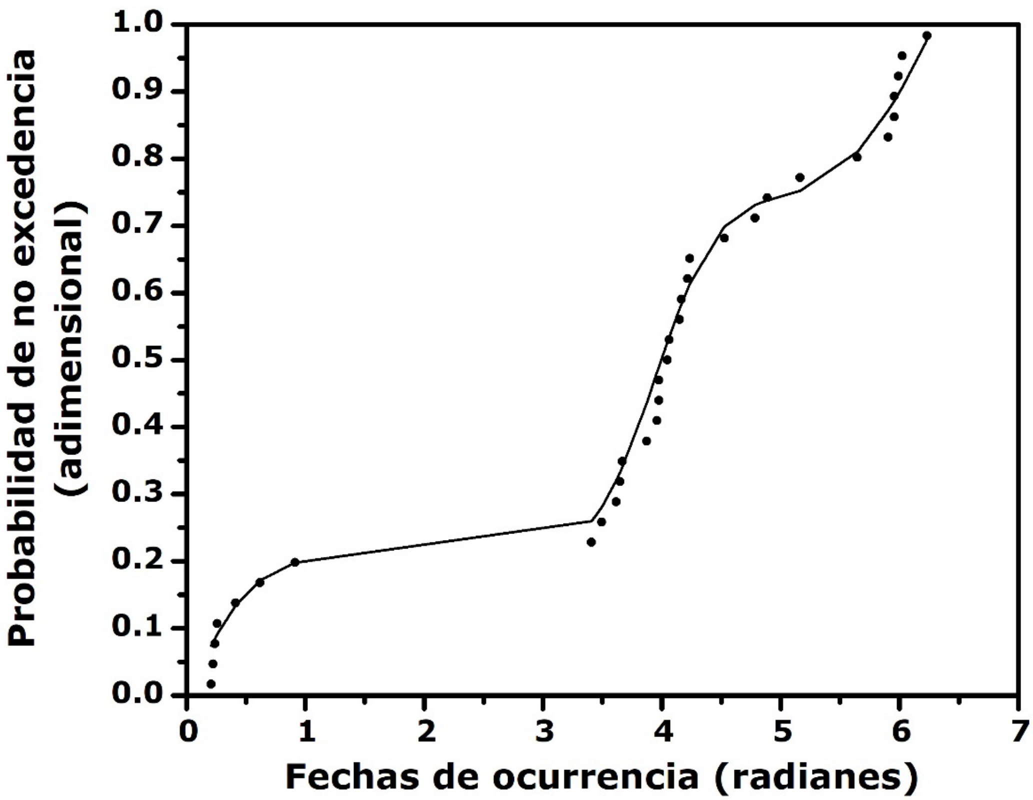

Figure 5. Type 3 fit of the von Mises distribution to the annual floods occurrence dates recorded at the San Francisco hydrometric station. Abscissa exis: Occurrence dates (radians); ordinate axis: non-exceedance probability (adimensional).

For the Huites hydrometric station, with the adopted and cited initial values of w_i, μ_i and κ_i, the following summary of optimal parameters in Table 9 is obtained, after 1 459 evaluations of the objective function (FO).

Figure 6 shows the contrast of the 51 empirical and theoretical non-exceedance probabilities achieved with the type 3 fit of the dvM.

Table 9. Results of the Complex algorithm for the type 3 adjustment at the Huites hydrometric station of the Hydrological Region No. 10 (Sinaloa), Mexico.

Parameters	Value	No. month	Acumulated		Increments	
			$F_E(x)$	$F_T(x)$	$F_E(x)$	$F_T(x)$
FO initial	0.04912	1	0.1569	0.1610	0.157	0.161
FO final	0.00292	2	0.2353	0.2321	0.078	0.071
w_1	0.17860	3	0.2549	0.2461	0.020	0.014
μ_1	0.40666	4	0.2549	0.2486	0.000	0.003
κ_1	5.38817	5	0.2549	0.2542	0.000	0.006
w_2	0.57650	6	0.2549	0.2798	0.000	0.026
μ_2	4.15254	7	0.3725	0.3701	0.118	0.090
κ_2	3.01271	8	0.5686	0.5487	0.196	0.179
w_3	0.23010	9	0.6863	0.7183	0.118	0.170
μ_3	6.18776	10	0.8235	0.7987	0.137	0.080
κ_3	6.97684	11	0.8431	0.8539	0.020	0.055
Suma de w_j	0.98530	12	1.0000	0.9987	0.157	0.145

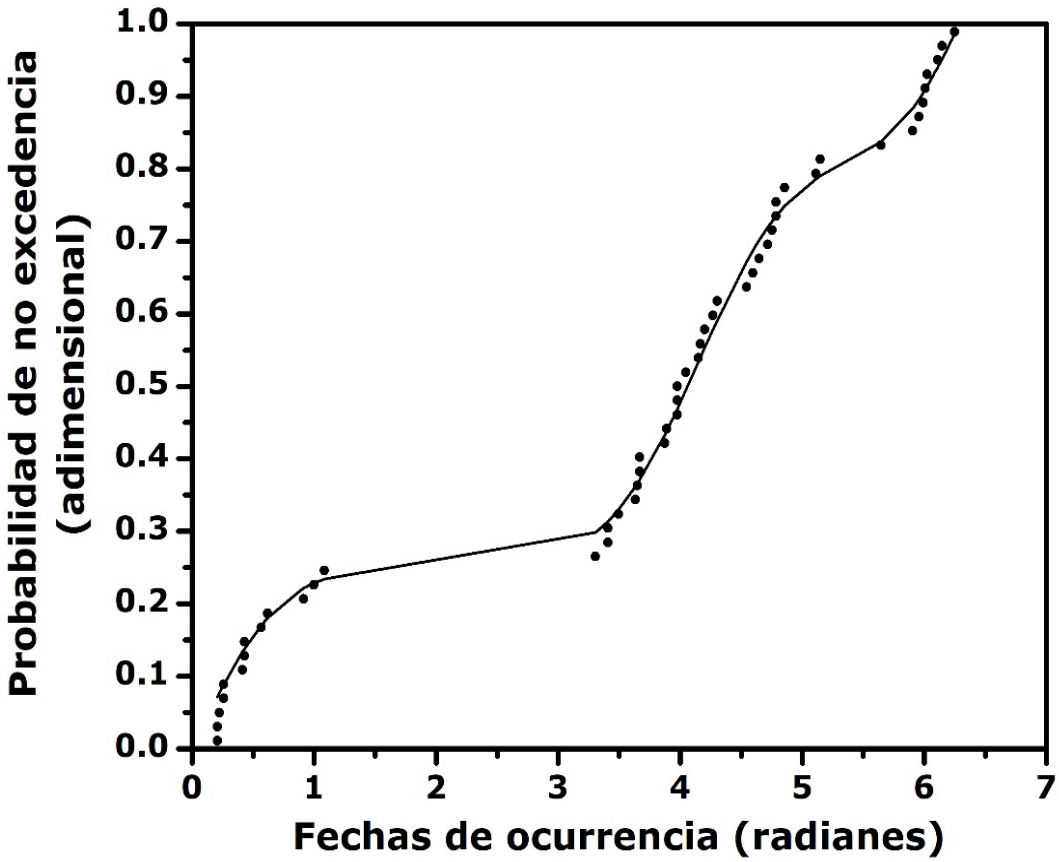

Figure 6. Type 3 fit of the von Mises distribution to the annual floods occurrence dates recorded at the Huites hydrometric station. Abscissa exis: Occurrence dates (radians); ordinate axis: non-exceedance probability (adimensional).

For the Jaina hydrometric station, Table 10 shows its 56 years of maximum annual flows and dates of occurrence.

Table 10. Flows of the annual floods and their dates of occurrence at the Jaina hydrometric station of Hydrological Region No. 10 (Sinaloa), Mexico.

No.	Q_{max} (m³/s)	Month	Day	No.	Q_{max} (m³/s)	Month	Day
1	2065	Oct	2	29	357	Ago	24
2	6991	Dec	9	30	1109	Oct	27
3	580	Feb	28	31	932	Nov	24
4	714	Oct	9	32	1349	Feb	22
5	747	Aug	10	33	680	Ago	31
6	771	Dec	4	34	488	Sep	4
7	693	Oct	8	35	900	Oct	24
8	2614	Jan	26	36	791	Ago	31
9	2336	Jan	13	37	989	Sep	27
10	437	Dec	15	38	1620	Feb	5
11	594	Aug	22	39	400	Ago	18
12	546	Sep	17	40	2832	Oct	8
13	516	Aug	19	41	4440	Sep	30
14	1600	Jan	15	42	179	Sep	19
15	639	Jun	27	43	694	Ago	13
16	362	Aug	28	44	494	Ene	26
17	2232	Sep	21	45	416	Oct	2
18	616	Aug	18	46	518	Ago	13
19	2003	Jan	12	47	105	Ago	23
20	795	Oct	31	48	227	Ago	20
21	1137	Oct	5	49	638	Dic	29

No.	Q_{max} (m³/s)	Month	Day	No.	Q_{max} (m³/s)	Month	Day
22	1226	Dec	12	50	309	Dic	23
23	454	Jul	29	51	372	Ene	17
24	650	Sep	2	52	216	Sep	23
25	958	Aug	15	53	199	Ago	21
26	900	Dec	17	54	174	Jul	18
27	1338	Sep	14	55	169	Nov	12
28	340	Jul	17	56	713	Sep	3

For the data in Table 10 from the Jaina gauging station and the adopted and cited initial values of w_i, μ_i and κ_i, define the following summary of optimal parameters from Table 11, after 1 076 evaluations of the objective function (FO). Figure 7 shows the contrast of the 56 empirical and theoretical non-exceedance probabilities achieved with the type 3 fit of the dvM.

Tecnología y Ciencias del Agua

OPEN ACCESS

Check for updates

Table 11. Results of the Complex algorithm for the type 3 fit at the Jaina hydrometric station.

Parameters	Value	No. Month	Acumulated		Increments	
			$F_E(x)$	$F_T(x)$	$F_E(x)$	$F_T(x)$
FO iniTial	0.02575	1	0.1071	0.1130	0.107	0.113
FO final	0.00291	2	0.1607	0.1500	0.054	0.037
w_1	0.16850	3	0.1607	0.1578	0.000	0.008
μ_1	0.10894	4	0.1607	0.1596	0.000	0.002
κ_1	3.52386	5	0.1607	0.1621	0.000	0.003
w_2	0.68740	6	0.1786	0.1777	0.018	0.016
μ_2	4.25620	7	0.2321	0.2571	0.054	0.079
κ_2	3.68355	8	0.5000	0.4693	0.268	0.212
w_3	0.12410	9	0.6786	0.7083	0.179	0.239
μ_3	6.21950	10	0.8393	0.8225	0.161	0.114
κ_3	8.27296	11	0.8750	0.8787	0.036	0.056
Suma de w_j	0.98000	12	1.0000	0.9987	0.125	0.120

Tecnología y Ciencias del Agua
OPEN ACCESS
Check for updates

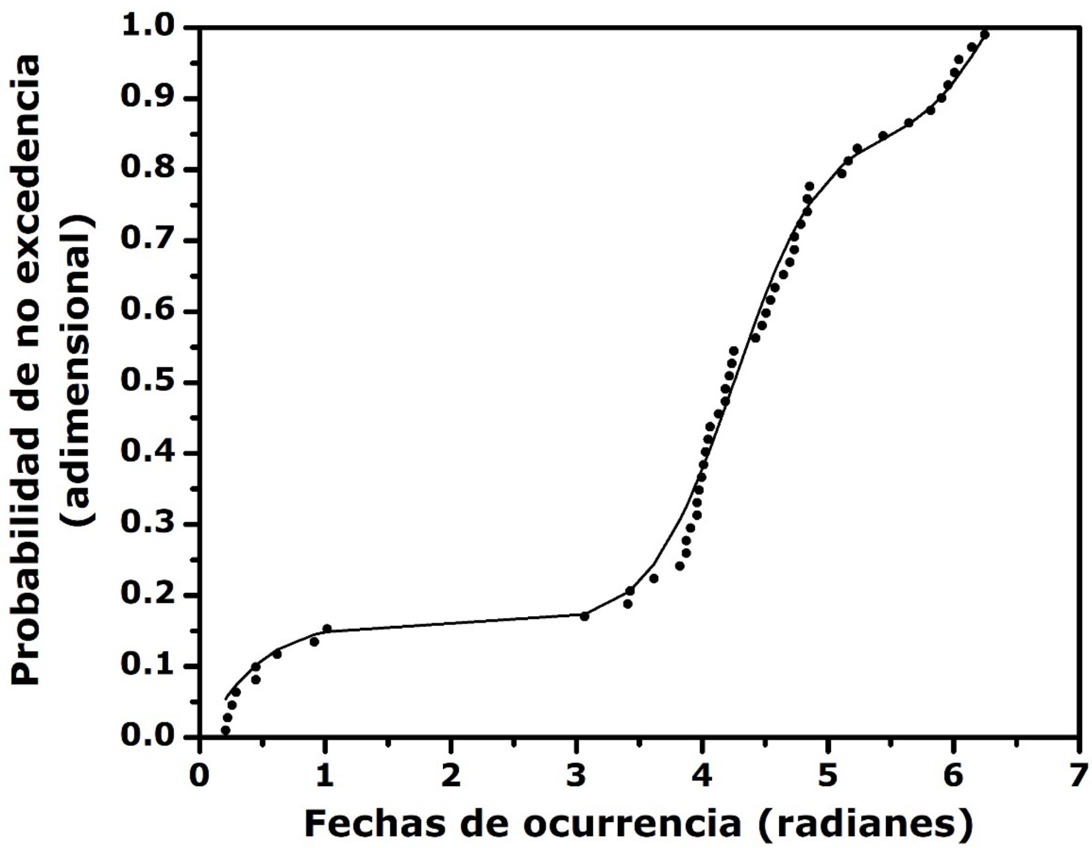

Figure 7. Monthly histogram and type 3 fit of the von Mises distribution to the annual floods occurrence dates recorded at the Jaina hydrometric station. Abscissa exis: Occurrence dates (radians); ordinate axis: non-exceedance probability (adimensional).

Tecnología y Ciencias del Agua

OPEN ACCESS

Check for updates

General remarks

In Hydrological Region No. 10 (Sinaloa), Mexico, studied, the size of the basin and the location of its hydrometric stations define the type of adjustment that must be made to probabilistically characterize the dates of its annual floods occurrence. Large mountainous basins with areas greater than 8 000 km^2 present a bimodal regime, with type 3 or mixed fit, such as: Huites, San Francisco, Santa Cruz and Jaina.

Intermediate basins with areas greater than 5 000 km^2 have a broad unimodal regime that accepts a type 1 or standard adjustment. Such is the case of Palo Dulce, Ixpalino, La Huerta and Chinipas. Finally, all the basins of the coastal plain with basin areas less than 2 000 km^2 exhibit a concentrated unimodal regime with type 2 or local fit.

In relation to the characterization of the dates of occurrence of the floods of this hydrological region, it is considered important to continue its study; for example, seeking to relate their regimes (standard, local and mixed) with the cold fronts, cyclonic and convective precipitation.

Conclusions

The *von Mises distribution* has been described in detail, which allows the probabilistic characterization of the of annual floods occurrence dates. Such dates are analyzed as circular data and their mean direction and seasonal index statistics define the two fit parameters of such distribution: its mode (μ) and its concentration parameter (κ). The von Mises distribution is symmetric and is considered the equivalent of the Normal distribution, for vectors or data with direction.

When the dates of occurrence of the annual floods occur throughout the year and also show a single concentration or *mode*, the von Mises distribution is fitting in a *standard* way with the maximum likelihood method, which establishes equations for μ and κ. The quality of fit achieved is measured by the sum of the differences between theoretical and empirical probabilities squared (SDPC). Empirical probabilities were assigned based on the Gringorten formula.

When the dates of occurrence of the annual floods are concentrated in one season of the year, the fit of the von Mises distribution was carried out using the Rosenbrock algorithm, which minimized the SDPC objective function, to find the optimal parameters μ and κ. This fitting named *local*.

Finally, when the dates of occurrence of the annual floods show two modes, the probabilistic characterization of such dates was sought with a mixture of three von Mises distributions, whose nine optimal fit parameters were obtained through the Complex algorithm, by minimizing the objective function SDPC, whose empirical probabilities were the dates of occurrence that occurred in the twelve months of the year. This setting was designated *mixed*.

In Hydrological Region No. 10 (Sinaloa), Mexico, which was studied, its large mountainous basins (> 8 000 km^2) present a *bimodal* regime of dates of occurrence of their annual floods. On the other hand, in the small basins (< 2 000 km^2), the dates of occurrence of their annual floods are concentrated in one season of the year. The basins, whose dates of occurrence of their annual floods occur throughout the year, are those of intermediate size and mountainous.

Acknowledgements

The author thanks the anonymous referees C and D for their comments and suggested corrections, which allowed to eliminate errors of omission and improve the general writing; in addition, to specify the scope of the study.

References

Aldama, A. A., Ramírez, A. I., Aparicio, J., Mejía-Zermeño, R., & Ortega-Gil, G. E. (2006). *Seguridad hidrológica de las presas en México.* Jiutepec, México: Instituto Mexicano de Tecnología del Agua.

Box, M. J. (1965). A new method of constrained optimization and a comparison with other methods. *Computer Journal*, 8(1), 42-52.

Bunday, B. D. (1985). Theme 6.2. The Complex method. In: *Basic optimisation methods* (pp. 98-106). London, UK: Edward Arnold Publishers, Ltd.

Burn, D. H. (1997). Catchment similarity for regional flood frequency analysis using seasonality measures. *Journal of Hydrology*, 202(1-4), 212-230. DOI: 10.106/S0022–1694(97)00068–1

Campos-Aranda, D. F. (2003). Capítulo 7. Integración numérica y Capítulo 9. Optimización numérica (pp. 137-153 y 172-211). En: *Introducción a los métodos numéricos: software en Basic y aplicaciones en hidrología superficial*. San Luis Potosí, México: Editorial Universitaria Potosina.

Campos-Aranda, D. F. (2014). Análisis regional de frecuencia de crecientes en la Región Hidrológica No. 10 (Sinaloa), México. 1: índices de estacionalidad y regiones de influencia. *Agrociencia*, 48(2), 147-158.

Campos-Aranda, D. F. (2017). Definición de tres épocas de crecientes utilizando estadísticos direccionales. *Tecnología y ciencias del agua*, 8(1), 155-165.

Carta, J. A., Bueno, C., & Ramírez, P. (2008). Statistical modelling of directional wind speeds using mixtures of von Mises distributions: Case study. *Energy Conversion and Management*, 49(5), 897-907. DOI: 10.1016/j.enconman.2007.10.017

Chen, L., Guo, S., Yan, B., Liu, P., & Fang, B. (2010). A new seasonal design flood method based on bivariate joint distribution of flood magnitude and date of occurrence. *Hydrological Sciences Journal*, 55(8), 1264-1280. DOI: 10.1080/02626667.2010.520564

Chen, L., Singh, V. P., Guo, S., Fang, B., & Liu, P. (2013). A new method for identification of flood seasons using directional statistics. *Hydrological Sciences Journal*, 58(1), 28-40. DOI: 10.1080/02626667.2012.743661

Cunderlik, J. M., Ouarda, T. B. M. J., & Bobée, B. (2004). Determination of flood seasonality from hydrological records. *Hydrological Sciences Journal*, 49(3), 511-526. DOI: 10.1623/hysj.49.3.511.54351

Davis, P. J., & Polonsky, I. (1972). Chapter 25. Numerical interpolation, differentiation and integration. In: Abramowitz, M., & Stegun, I. A. (eds.). *Handbook of mathematical functions*, 9th print. (pp. 875-926). New York, USA: Dover Publications.

Durrans, S. R., Eiffe, M. A., Thomas Jr., W. O., & Goranflo, H. M. (2003). Joint seasonal/annual flood frequency analysis. *Journal of Hydrologic Engineering*, 8(4), 181-189. DOI: 10.1061/(ASCE)1084-0699(2003)8:4(181)

Escalante-Sandoval, C. A., & Reyes-Chávez, L. (2002). Capítulo 9. Análisis conjunto de eventos hidrológicos. En: *Técnicas estadísticas en hidrología* (pp. 203-246). México, DF, México: Facultad de Ingeniería de la UNAM.

Genest, C., & Chebana, F. (2017). Copula Modeling in Hydrologic Frequency Analysis. In: Singh, V. P. (ed.). *Handbook of applied hydrology* (2nd ed.) (pp. 30.1-30.10). New York, USA: McGraw-Hill Education.

Khedun, C. P., Singh, V. P., & Byrd, A. R. (2019). Joint probability of extreme streamflow and its day of occurrence. *Journal of Hydrologic Engineering*, 24(8), 06019005:1-8. DOI: 10.1061/(ASCE)HE.1943-5584.0001813

Kite, G. W. (1977). *Frequency and risk analyses in hydrology*. Fort Collins, USA: Water Resources Publications.

Kottegoda, N. T., & Rosso, R. (2008). Theme 3.5. Copulas. In: *Applied statistics for civil and environmental engineers* (2nd ed.) (pp. 154-157). Oxford, UK: Blackwell Publishing.

Kuester, J. L., & Mize, J. H. (1973). Chapter 9. Multivariable unconstrained methods. III. ROSENB algorithm. In: *Optimization techniques with Fortran* (pp. 320-330). New York, USA: McGraw-Hill Book Co.

Metcalfe, A. V. (1997). Theme 2.4.4. Vector variables and von Mises distribution. In: *Statistics in civil engineering* (pp. 31-32). London, UK: Arnold Publishers.

Meylan, P., Favre, A. C., & Musy, A. (2012). *Predictive hydrology. A frequency analysis approach*. Boca Raton, USA: CRC Press.

Nieves, A., & Domínguez, F. C. (1998). Sección 6.2. Cuadratura de Gauss. En: *Métodos numéricos. aplicados a la ingeniería* (pp. 416-425). México, DF, México: Compañía Editorial Continental.

Olver, F. W. J. (1972). Chapter 9. Bessel functions of integer order. In: Abramowitz, M., & Stegun, I. A. (eds.). *Handbook of Mathematical Functions* (9th print) (pp. 355-436). New York, USA: Dover Publications.

Ramírez-Orozco, A. I., Gutiérrez-López, A., & Ruiz-Silva, H. L. (2009). Análisis de la ocurrencia en el tiempo de los gastos máximos en México. *Ingeniería hidráulica en México*, 24(1), 115-124.

Rao, A. R. & Hamed, K. H. (2000). *Flood frequency analysis*. Boca Raton, USA: CRC Press.

Rosenbrock, H. H. (1960). An automatic method of finding the greatest or least value of a function. *Computer Journal*, 3(3), 175-184.

Stedinger, J. R., Vogel, R. M., & Foufoula-Georgiou, E. (1993). Chapter 18: Frequency analysis of extreme events. In: Maidment, D. R. (ed.). *Handbook of Hydrology* (pp. 18.1-18.66). New York, USA: McGraw-Hill Inc.

Stedinger, J. R. (2017). Flood frequency analysis. In: Singh, V. P. (ed.). *Handbook of Applied Hydrology* (2nd ed.) (pp. 76.1-76.8). New York, USA: McGraw-Hill Education.

Teegavarapu, R. S. V., Salas, J. D., & Stedinger, J. R. (2019). *Statistical analysis of hydrologic variables*. Reston, USA: American Society of Civil Engineers.

Yue, S., & Rasmussen, P. (2002). Bivariate frequency analysis: Discussion of some useful concepts in hydrological application. *Hydrological Processes*, 16(14), 2881-2898. DOI: 10.1002/hyp.1185

Zhang, L., & Singh, V. P. (2006). Bivariate flood frequency analysis using the Copula method. *Journal of Hydrologic Engineering*, 11(2), 150-164. DOI: 10.1061/(ASCE)1084-0699(2006)11:2(150)

Zhang, L., & Singh, V. P. (2019). Chapter 3. Copulas and their properties. In: *Copulas and their applications in water resources engineering* (pp. 62-122). Cambridge, UK: Cambridge University Press.